Time Averaging Meets Labor Supplies of Heckman, Lochner, and Taber

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Abstract

We add endogenous career lengths to the Heckman, Lochner, and Taber (1998a) (HLT) model with its credit markets and within-period labor supply indivisibilities, all of which are essential features of Ljungqvist and Sargent (2006) "time-averaging." A benchmark social security system puts all workers at corner solutions of their retirement decisions. That lets our model reproduce most outcomes in HLT's model with its inelastic labor supply and mandatory retirement date for all types of workers. Eight types of workers are indexed by pairs of innate abilities and choices of education levels. Tax and social security arrangements can dislodge some types of agents from those corners, bringing associated changes in equilibrium prices, college enrollments, and on-the-job human capital accumulations. A reform that links social security benefits to age but not to employment status eliminates an implicit tax on working beyond age 65. High tax rates with revenues returned lump-sum keep agents off corner solutions, raising the aggregate labor supply elasticity and threatening to bring about a "dual labor market" in which many people decide not to supply labor.

Key words: Time averaging, labor supply elasticity, retirement, taxation, Laffer curve, social security reform.

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1 Introduction

This paper adds individual career length choices and a social security system to the life-cycle model of Heckman, Lochner, and Taber (1998a, henceforth HLT). We use our model to study consequences of social security reforms and a tax reform studied by Prescott (2002). While we adopt almost all of HLT's model components, we drop HLT's assumption of an exogenous retirement age of 65 and instead add a social security system with alternative rules governing eligibility for retirement benefits. Under rules like ones that governed the US system until recently, workers find themselves at corner solutions for choices of career lengths that induce them to retire at age 65. Outcomes in this version of our model closely resemble HLT's. But under new rules that let people receive social security benefits after 65 without actuarial loss of benefits even if they continue to work, some workers, especially highly skilled, highly educated ones, extend working careers beyond age 65. Our tax reform with revenues returned lump-sum also keeps agents off corner solutions about when to retire, which affects their decisions about whether to attend college and how much human capital to accumulate while working, in turn generating substantial effects on individual and aggregate labor supply elasticities.[1]

Like HLT's model, ours is a perfect foresight model of overlapping generations of finitely lived agents who smooth consumption across time periods partly by saving and dissaving a risk-free bond. Eight types of agents are indexed by pairs of innate abilities (there are four ability levels) and education levels (there are two education levels, high school and college). Each type has a Ben-Porath (1967) technology that constrains how it accumulates human capital. An aggregate production function takes as inputs physical capital and two types of human capital – some owned by college workers, others owned by high school educated workers. Each period, a decision to work is dichotomous – a person either works or doesn't work. Life-cycle consumption smoothing, freedom to choose how many periods to work, and an indivisibility of within-period labor supplies are essential features of the Ljungqvist and Sargent (2006) "time-averaging" component of our model that lets endogenous retirement ages unleash forces that shape aggregate labor supply elasticities.

At least since Lucas and Rapping (1969), macroeconomists have inferred high labor

[1] Along with HLT, many other macroeconomic applications of life-cycle models dating back at least to Auerbach and Kotlikoff's (1987) quantitative policy analysis have also assumed an exogenous retirement age. Another ingredient of the low aggregate labor supply elasticity was HLT's model assumption of inelastic labor supplies before retirement, which HLT (p. 9) justified by noting that "[e]stimates of intertemporal substitution in labor supply estimated on annual data are small, so ignoring labor supply decisions will not greatly affect our analysis."

supply elasticities from employment fluctuations over business cycles. Prescott rationalized those high elasticities in two versions of his Nobel prize lecture, first in Prescott (2005) when he appealed to the employment lotteries model of Rogerson (1988), then in Prescott (2006b) when he switched to the "time-averaging" model. In the context of the present paper, it is noteworthy that Prescott changed horses midstream when writing his Nobel prize lecture.

In the original published version of his 2004 Nobel lecture, Prescott (2005, p. 385) appealed to an aggregation theory of Rogerson (1988) that he praised as "every bit as important as the one giving rise to the aggregate production function." Rogerson obtained a high aggregate labor supply elasticity by combining (i) a representative family consisting of a continuum of *ex ante* identical workers who can supply either 0 or 1 unit of labor in any period, with (ii) a planner who each period chooses a fraction of family members to send to work, uses employment lotteries to decide who works and who doesn't, then allocates the same consumption to all family members.[2]

After discussing Ljungqvist and Sargent (2006) at the 2006 NBER macro annual conference (see Prescott (2006a)), Prescott (2006b) revised his original Nobel lecture to incorporate their "time-averaging" model.[3] The "time-averaging" theory retains Rogerson's assumption about indivisible labor, but drops his assumptions that a planner uses employment lotteries to assign a fraction of unlucky family members to work each period, then awards the same consumption to all family members each period. In the time-averaging model, each worker chooses a fraction of its own life to work and smooths consumption over time by saving and dissaving a risk-free bond. Each period, a person decides whether or not to work that period. In a continuous-time, non-stochastic life-cycle incomplete-market economy that retains indivisible labor component, Ljungqvist and Sargent (2006) deduced the same individual (expected) utilities, aggregate allocation and high aggregate labor supply elasticity that prevail in the corresponding version of Rogerson's economy with employment lotteries.[4]

[2] Rogerson showed how to support the same allocation in a competitive equilibrium with employment lotteries. So long as an equilibrium employment-to-population ratio is less than one, a high aggregate labor supply elasticity emerges. Because they saw no micro evidence for them, skeptics expressed doubts about the employment lotteries and the social insurance insurance in Rogerson's theory. Browning, Hansen, and Heckman (1999, p. 602) wrote that the "employment allocation mechanism strains credibility and is at odds with the micro evidence on individual employment histories."

[3] In a section titled "The Life Cycle and Labor Indivisibility," Prescott (2006b) adopted and extended the Ljungqvist and Sargent (2006) analysis to include an intensive margin in agents' labor supply. Prescott (2006a) had welcomed their time-averaging model as providing a more plausible micro-foundation for a high aggregate labor supply elasticity.

[4] Independently, Chang and Kim (2006) discovered a high aggregate labor supply elasticity in simulations of a stochastic Bewley model with incomplete markets and indivisible labor. Their agents optimally alternate between periods of work and leisure (they "time average") to allocate consumption and leisure over their

We bring the time-averaging mechanism into our model by dropping HLT's exogenous retirement age and instead letting people choose when to retire. To create a version of our model in which everyone nevertheless chooses to retire at 65, we add a social security system that imposes high implicit taxes on people who choose to work after they first become eligible to receive retirement benefits. Those high implicit taxes put everyone at kinks of boundaries of their budget sets that are created by that implicit tax.[5]

This structure mimics outcomes found by HLT. Using it as a baseline, we study a social security reform that lets people receive social security benefits after the official retirement age, regardless of whether they choose to retire. This is our representation of a reform in U.S. social security that has recently been put in place. After the reform, agents who delay claiming social security payments lose nothing as measured by the actuarial value of lifetime benefits. By removing the kink in workers' budget constraints, the reform dislodges people from a corner at the official retirement age under the old social security arrangement.[6]

We conduct another policy experiment that studies consequences of increases in the labor income tax rate above its value in our baseline economy. Prescott (2002) stressed that effects of labor tax rate increases on the aggregate labor supply depend on how a government spends resulting changes in tax revenues. A combination of income and substitution effects on labor supplies brings this dependence. If the government spends those revenues on very good substitutes for private consumption, it provokes a high labor supply elasticity. If the government spends those revenues on goods that are poor substitutes for private consumption, it provokes a low aggregate labor supply elasticity. Income and substitution effects like Prescott's are also active in our model. In our baseline economy, effects of tax rate increases also depend on whether workers are at corner solutions for career lengths at

infinite lifespans.

[5]We justify our assumption that social security benefits not collected after age 65 are a complete loss to a worker by referring to Schulz (2001, pp. 141-2), who described how this was actually the situation in the U.S. social security system between 1950 and 1972, after the 1950 suspension of an earlier provision of a 1 percent increase in benefits for each year of delayed retirement. After 1972, a delayed retirement credit was reintroduced, but it is only with rules that recently became effective that the compensation is high enough for there to be no loss in the actuarial value of a worker's lifetime benefits.

[6]Corner solutions to career lengths and their undoing are parts of the Ljungqvist and Sargent (2011) forecast of a "labor supply elasticity accord." On the one hand, both North America and Europe had historically instituted social security programs with implicit tax wedges that implied corner solutions to career lengths for primary workers at the official retirement ages, which led researchers like those in footnote 1 to infer low labor supply elasticities. On the other hand, due to recent developments including social security reforms, workers are moved off corner solutions and having instead to choose interior solutions to career lengths. In such a "time-averaging" environment, Prescott's above view of a high aggregate labor supply elasticity might ultimately be proven correct.

the official retirement age. In our economy under the social security reform, effects of tax rate increases also depend on whether people choose to work even after the efficiency of their human capital starts to decline pretty rapidly.

While our findings reaffirm some insights that emerged from simple partial-equilibrium structures of Ljungqvist and Sargent (2006, 2014), our model unleashes additional forces that must be reconciled by equilibrium prices and quantities of human capital accumulated by agents of different ability types and education levels. Section 2 describes our model while also highlighting and explaining how it differs from the HLT framework. In Section 3, we calibrate parameters, some borrowed from HLT, others "backed out" from HLT, and yet other parameters, new to our model, that we set to be compatible with HLT's parameter choices. By verifying that our parameter settings enable our model to mimic HLT's findings, this section sets the stage for the policy experiments in subsequent sections. Section 4 analyzes the social security reform. Section 5 analyzes the tax-and-transfer policies. Section 6 analyzes a combination of both policies. Section 7 computes aggregate labor supply elasticities for these policy experiments. Section 8 studies implications of our policy experiments on various measures of inequality. Section 9 contains concluding remarks about how adjustments of prices and quantities of physical and human capital stocks to new tax and transfer policies have possibly unintended consequences for the distributions of labor incomes and welfare across workers. Such findings pose challenges for economic policy making. A post scriptum in Section 10 offers additional insights about time averaging when comparing some of our findings to those of Fan, Seshadri, and Taber (2024) who independently analyzed endogenous career lengths in a Ben-Porath human capital model.

2 Model

2.1 Primitives

We keep our model as close as possible to the framework of HLT, while endogenizing career lengths and introducing a pay-as-you-go social security system. Auxiliary changes of HLT's primitives include our adding declining efficiency units of human capital at older ages and our use of an alternative shortcut designed to target capital-output ratio. Accompanying text boxes display all differences in primitives between our model and HLT's.

We study an overlapping generations model in discrete time at an annual frequency. Each person lives inside the model from age $\underline{\eta} = 18$ to age $\overline{\eta} = 80$. A cohort of 80-year olds that exits the model each year is replaced by an equal-sized new cohort of 18-year olds who enter the model with high school degrees and who either start to work immediately as high school graduates or choose to attend four-year college and then work as college graduates. Workers face within-period labor supply indivisibilities: in any period they either work (or attend college) full time, $\omega = 1$, or do not work (or attend college), $\omega = 0$, with leisure equal to $1 - \omega$. An agent orders streams of a single consumption good and leisure by standard time-additively preferences with subjective discount factor δ. Following Ljungqvist and Sargent (2014), utility from consuming C and supplying labor $\omega \in \{0, 1\}$ within a period is

$$U(C, \omega) = \frac{C^{1-\gamma}}{1-\gamma} \max\{1 - \omega, B\omega\}\,, \qquad \gamma, B > 0, \tag{1}$$

where for $0 < \gamma < 1$ ($\gamma > 1$), we require $0 < B < 1$ ($B > 1$) in order to make utility decrease in labor supply. By virtue of L'Hôpital's rule, utility of consumption converges to a logarithmic function as γ approaches one; so for $\gamma = 1$, the utility function becomes

$$\tilde{U}(C, \omega) = \log(C) - B\omega\,, \qquad B > 0. \tag{2}$$

Preferences (1) and (2) are consistent with balanced growth and have a constant intertemporal elasticity of substitution in consumption equal to $1/\gamma$.

In contrast, HLT assume inelastic labor supply, $\omega = 1$, up and until a mandatory retirement age 65. HLT attach no disutility to working, so their preferences agree with (1) and (2), except that they remove the multiplicative factor $\max\{1 - \omega, B\omega\}$ in (1) and the additive term $-B\omega$ in (2).

Agents are heterogeneous with respect to their innate abilities and costs of attending college. When a cohort enters the economy with high school degrees at age 18, it is divided into four equal-sized ability groups indexed by $\theta \in \{1, 2, 3, 4\}$. All members of the same ability group share identical endowments of human capital, technologies for on-the-job human capital accumulation, and probability distributions that govern an idiosyncratic nonpecuniary cost ϵ of attending college. The cost ϵ is drawn from a normal distribution with standard deviation σ and ability-specific mean μ_θ. Given an ability θ and a realization ϵ of cost, an agent makes an irrevocable decision either immediately to work as a high school graduate (this is schooling choice $S = 1$) or to attend four-year college at an annual tuition cost of ζ

and, after graduating, to work as a college graduate (this is schooling choice $S = 2$).

An agent of ability θ who makes schooling choice S has an initial endowment $H^S(\theta)$ units of S-specific human capital. While on the job, the agent can augment his human capital by diverting time from working to investing in human capital, with total time available per period normalized to one. Thus, given a human capital stock H^S_n at age n, an agent who made schooling choice S can use a fraction I of his time to acquire age $n+1$ human capital according to a Ben-Porath (1967) technology

$$H^S_{n+1} = A^S(\theta)\, I^{\alpha_S} \left(H^S_n\right)^{\beta_S} + H^S_n \,, \tag{3}$$

where $A^S(\theta) > 0$, $0 < \alpha_S < 1$ and $0 \leq \beta_S \leq 1$.[7] Hence, the ability-specific talents of an agent in ability group θ are encoded in an initial human capital endowment $H^S(\theta)$ and a multiplicative factor $A^S(\theta)$ of the human capital technology. There is a pair $\left(H^S(\theta), A^S(\theta)\right)$ for each schooling choice $S = 1, 2$. Parameters (α_S, β_S) are common to high school ($S = 1$) and college graduates ($S = 2$), respectively. To describe how human capital gets converted into age-dependent efficiency units, let

$$e(n) = \frac{1}{1 + \exp(\phi_1(n - \phi_2))} \leq 1 \tag{4}$$

be a multplicative factor that translates the human capital stock of an agent of age n into efficiency units. Thus, given a schooling choice S, an agent of age n with human capital stock H^S_n has $e(n)H^S_n$ efficiency units that can be employed in the goods production technology (5) below. But note that the human capital technology (3) is not affected by efficiency units.

> While HLT assume no depreciation of human capital, we introduce the age-dependent mapping (4) of human capital into efficiency units in order to attenuate labor supply responses in old age in some of our policy experiments.

The aggregate production function exhibits constant returns to scale,

$$F(\bar{H}^1, \bar{H}^2, \bar{K}) = a_3 \left((1 - a_2) \left[a_1 \left(\bar{H}^1\right)^{\rho_1} + (1 - a_1) \left(\bar{H}^2\right)^{\rho_1} \right]^{\rho_2/\rho_1} + a_2 \bar{K}^{\rho_2} \right)^{1/\rho_2} . \tag{5}$$

[7] HLT (p. 19) assume no depreciation of human capital so as to be "consistent with the lack of any peak in life-cycle wage–age profiles reported in the literature."

Inputs are the aggregate physical capital stock $\bar{K}$ and the aggregate human capital $\bar{H}^S$ supplied by high school ($S = 1$) and college ($S = 2$) workers, respectively. Notice that the quantity $\bar{H}^S$ excludes time and human capital that is used to create additional human capital for next period. Elasticities of substitution are $(1 - \rho_1)^{-1}$ between high school and college human capital and $(1-\rho_2)^{-1}$ between physical capital and composite human capital, respectively. Taking a limit as parameter ρ_2 approaches zero, the technology becomes

$$\tilde{F}(\bar{H}^1, \bar{H}^2, \bar{K}) = a_3 \left[a_1 \left(\bar{H}^1\right)^{\rho_1} + (1 - a_1) \left(\bar{H}^2\right)^{\rho_1}\right]^{(1-a_2)/\rho_1} \bar{K}^{a_2}, \tag{6}$$

which is a Cobb-Douglas production function with inputs being the physical capital stock and a CES aggregate of the two types of human capital, where the physical capital share is a_2 and the labor share $1 - a_2$. Output can be allocated to private consumption, government consumption, or additional physical capital for next period.[8] Government consumption G enters neither agents' preferences nor technologies for producing goods and human capital.

The government taxes, spends, and runs a pay-as-you-go social security system. It levies flat-rate taxes τ_l and τ_k on households' labor and capital incomes, respectively. The tax rate on capital income is also the rate at which interest expenses on borrowings are deductible from an agent's tax liability, so borrowers and lenders both face an effective interest rate equal to $(1 - \tau_k)r$, where r is the market interest rate. The baseline social security program consists of a payroll tax rate τ_p levied on all labor incomes, plus a social security benefit P paid to all agents of age $\eta_p = 65$ or older who are not working. The benefit P is taxed at the labor income tax rate τ_l. The government balances a consolidated budget constraint, including the social security program. Following HLT, that budget constraint is satisfied by adjusting government consumption G.

Since there are no aggregate shocks and all individual uncertainty is resolved when agents enter the economy, competitive equilibria are computed under perfect foresight. Agents can freely lend and borrow at a risk-free interest rate.[9] In a general equilibrium, net savings should equal the stock of physical capital. But since we want to target a particular capital-

[8] HLT assume no depreciation of physical capital. However, if we were to want to accommodate reductions in the capital stock within an equilibrium, we could simply adopt an assumption of reversible capital, e.g., by assuming that physical capital can one-for-one be converted into goods for consumption.

[9] HLT (p. 10) note the absence of "short-run credit constraints that are often featured in the literature on schooling and human capital accumulation. Our model is consistent with the evidence presented in [e.g., Cameron and Heckman (1998)] that long-run family factors correlated with income (the θ operating through $A^S(\theta)$ and $H^S(\theta)$) affect schooling, but that short-term credit constraints are not empirically important. ... The mechanism generating the family income–schooling relationship operates through family-acquired human capital and not credit rationing."

output ratio, we resort to the following modeling shortcut. We assume that agents who live inside our model hold only a share κ of the equilibrium capital stock and that the remaining share $1-\kappa$ is held by "investors" whom we do not explicitly model but whose capital income is also taxed by the government – an approach that retains interest rate endogeneity in model simulations.[10] Hence, these investors' net-of-tax payout each period is $(1-\tau_k)r(1-\kappa)\bar{K}$.

HLT have no social security system and require workers to retire at age 65. Our model has a social security system that might motivate workers to choose to retire at an official retirement age.
HLT used a lump-sum transfer from 65-year olds to 18-year olds in order to target a capital-output ratio. That approach becomes unworkable in our model with its pay-as-you-go social security system that depresses private savings: it would require very large such lump-sum transfer to target HLT's capital-output ratio. Therefore, we instead assume that a fraction κ of the capital stock is held by the agents who live inside the model. Another justification for using this approach as we switch from HLT's inelastic labor supply to our model of endogenous career length is that lump-sum transfers become a potent determinant of labor supplies, especially for low-ability agents with meager labor earnings prospects. By doing away with HLT's lump-sum transfers, we ensure that an auxiliary assumption designed to target a capital-output ratio does not unduly affect other equilibrium outcomes.

2.2 Choices

Agents first choose a schooling level, then a retirement age, and then paths for consumption, saving, and human capital investment. Let's work backwards.

Conditional on retiring at age $\hat{n}$, the value function of an employed agent of age $n < \hat{n}$, type θ, schooling level S, savings K, and human capital H satisfies the Bellman equation

[10]Because plain vanilla life-cycle models typically fail to explain observed high wealth inequalities and fall short of explaining levels of aggregate savings as measured by an economy's stock of physical capital, researchers have activated other features that affect wealth accumulation, including entrepreneurship and bequest motives (see e.g. De Nardi (2015) for a survey of the literature). Our assumption that agents who live inside the model hold only a fraction of the equilibrium capital stock acknowledges such features omitted from our framework. In contrast to earlier studies that use such an assumption to deduce a fixed interest rate at which the analyses are conducted (see e.g. Storesletten, Telmer, and Yaron (2004)), we keep the fraction κ of capital held by agents in the model fixed across policy experiments, but for one exception, and consequently have an endogenous interest rate. The exception is our perturbation of the model in Section 4 that removes the social security system, which calls for a recalibration of the fraction κ.

$$V_n^{\hat{n}}(H,K,S,\theta) = \max_{C,I,K',H'} \Big[U(C,1) + \delta V_{n+1}^{\hat{n}}(H',K',S,\theta)\Big] \tag{7}$$

subject to

$$C + K' = (1 + (1-\tau_k)r)K + (1-\tau_l-\tau_p)R^S e(n)(1-I)H, \tag{8}$$

$$H' = H + A^S(\theta)I^{\alpha_S}H^{\beta_S}, \tag{9}$$

where r is the interest rate and R^S is the rental rate on human capital of schooling level S. The value function for the same type of agent during retirement, $n \geq \hat{n}$, satisfies

$$V_n^{\hat{n}}(H,K,S,\theta) = \max_{C,K'} \Big[U(C,0) + \delta V_{n+1}^{\hat{n}}(H,K',S,\theta)\Big] \tag{10}$$

subject to

$$C + K' = (1 + (1-\tau)r)K + (1-\tau_l)\mathbb{1}(n \geq \eta_p)P, \tag{11}$$

where $\mathbb{1}(n \geq \eta_p)$ is an indicator function equal to 1 if $n \geq \eta_p$, and zero otherwise. In the final year $n = \overline{\eta}$, the value function on the right side of the Bellman equation is zero, $V_{\overline{\eta}+1}^{\hat{n}}(\cdot) = 0$, and maximization is subject to an additional constraint requiring the worker to be solvent, $K_{\overline{\eta}+1} \geq 0$.

During the first four years of life, an $S = 2$ agent who attends college confronts a consumption-saving decision. So at ages $n = \underline{\eta} + j$ for $j = 0, 1, 2, 3$, the agent's optimization problem (7)–(9) is modified as follows. Budget constraint (8) is altered to become

$$C + K' + \zeta = (1 + (1-\tau_k)r)K, \tag{12}$$

where college tuition ζ is added to the left side, and there is no labor income on the right side. Furthermore, we impose $I = 0$ during college attendance so that by law of motion (9), the agent's college human capital at graduation is equal to $H_{\underline{\eta}+4}^2 = H^2(\theta)$, i.e., the endowment contingent on having graduated from college. Instead of earning a college degree, the agent could have begun working immediately at age $\underline{\eta}$ as a high school graduate with high school human capital equal to the endowment, $H_{\underline{\eta}}^1 = H^1(\theta)$. Every agent enters the economy with zero savings, $K_{\underline{\eta}} = 0$. Conditional on a retirement age $\hat{n}$, we can use a shooting algorithm to solve this part of the problem.

Conditional on a schooling choice S, an optimal retirement age $\hat{n}$ solves

$$\hat{V}^S(\theta) = \max_{\hat{n}} \Big\{ V_{\underline{\eta}}^{\hat{n}}(H^S(\theta), 0, S, \theta) :\ \hat{n} = \underline{\eta} + 4(S-1), \ldots, \overline{\eta} + 1 \Big\}, \tag{13}$$

where the choice $\hat{n} = \overline{\eta} + 1$ means that the agent never retires. Following HLT, a decision to attend college ($S = 2$) is a four-year commitment which, in our analysis, means that the agent cannot choose to retire until after graduation, $\hat{n} \geq \underline{\eta} + 4$.[11] Finally, an agent who has drawn a nonpecuniary college cost ϵ attends college if

$$\hat{V}^2(\theta) + \epsilon \geq \hat{V}^1(\theta), \tag{14}$$

where $\epsilon \sim N(\mu_\theta, \sigma)$. We let $p(S|\theta)$ denote the probability that an agent of ability θ makes schooling choice S.

Throughout the above description of an agent's optimization problem, we have proceeded under the assumption that it is optimal for an agent to front-load working, i.e., to work during a first part of life and to enjoy retirement during a second part. Five features ensure the optimality of such a career strategy. First, as in HLT, the calibration of our time averaging model is such that the equilibrium after-tax interest rate is greater than the subjective rate of discounting, $(1 - \tau_k)r > \delta^{-1} - 1$, and hence, it is optimal for an agent to generate labor income early in life in order to earn the high market interest rate on accumulated savings. Second, after learning his ability type θ and idiosyncratic nonpecuniary cost ϵ of attending college, an agent faces no further uncertainty; therefore, an optimal lifetime labor supply is deterministic. Third, the assumption that efficiency units of human capital diminish with age means that postponing labor supply brings lower ratios of efficiency units to human capital. Fourth, disadvantages of postponing labor supply are increased further under the social security system in our baseline economy in which all labor income after age 65 is subject to an extra implicit tax in the form of lost social security benefits that could have been collected if a worker had instead retired. Fifth, under HLT's and our assumption of frictionless credit markets, an agent can choose any lifetime consumption profile consistent with his present-value budget constraint; for example, an agent can mortgage his future social security benefits. That could be a good decision if lifetime labor supply calls for early retirement before the official retirement age.

[11] Like HLT, we assume a nonpecuniary random cost ϵ of attending college for four years; it is a cost if the realization of ϵ is negative, a benefit if the ϵ realization is positive.

2.3 Profit maximization

Competitive firms operate the production technology shown in equation (5). Skill prices and the return on capital satisfy the following first-order conditions:

$$R^1 = \frac{\partial F(\bar{H}^1, \bar{H}^2, \bar{K})}{\partial \bar{H}^1}, \tag{15}$$

$$R^2 = \frac{\partial F(\bar{H}^1, \bar{H}^2, \bar{K})}{\partial \bar{H}^2}, \tag{16}$$

$$r = \frac{\partial F(\bar{H}^1, \bar{H}^2, \bar{K})}{\partial \bar{K}}. \tag{17}$$

Recall that we have kept HLT's assumption of no depreciation of physical capital (see footnote 8).

2.4 Stationary equilibrium

We say that a parameterization is 'regular' if it supports a stationary equilibrium that satisfies two conditions: (i) all agents optimally front-load their labor supply, and (ii) conditional on a schooling choice, solutions to all agents' optimization problems are unique. We call such an equilibrium a *regular stationary equilibrium.*

Definition: A *regular stationary equilibrium* is a per-person allocation of consumption $\hat{C}_n(S, \theta)$, human capital $\hat{H}_n(S, \theta)$, fraction of time $\hat{I}_n(S, \theta)$ devoted to on-the-job human capital accumulation, and savings $\hat{K}_{n+1}(S, \theta)$, indexed by age $n = \underline{\eta}, \underline{\eta}+1, \underline{\eta}+2, \ldots, \overline{\eta}$, schooling $S = 1, 2$, and ability $\theta = 1, 2, 3, 4$, with associated retirement age $\hat{n}(S, \theta)$, and probability $p(S|\theta)$ that an agent of ability θ makes schooling choice S; aggregate quantities $\{\bar{K}, \bar{H}^1, \bar{H}^2\}$ of inputs in goods production; prices $\{r, R^1, R^2\}$; and government policy $\{\tau_l, \tau_k, \tau_p, \eta_p, P, G\}$ such that:

1. Given prices $\{r, R^1, R^2\}$ and government policy $\{\tau_l, \tau_k, \tau_p, \eta_p, P\}$, for each pair (θ, S) the allocation $\{\hat{C}_n(S, \theta), \hat{H}_n(S, \theta), \hat{I}_n(S, \theta), \hat{K}_{n+1}(S, \theta)\}_{n=\underline{\eta}}^{\overline{\eta}}$ and retirement age $\hat{n}(S, \theta)$ solve an agent's problem in (7) – (13).

2. For each ability group θ, the fraction of agents making schooling choice S is equal to $p(S|\theta)$, as determined by agents' decision rule on schooling in (14).

3. Prices $\{r, R^1, R^2\}$ are consistent with aggregate quantities $\{\bar{K}, \bar{H}^1, \bar{H}^2\}$ of inputs in goods production under perfect competition, i.e., prices satisfy (15) – (17).

4. The capital market clears:

$$\kappa\bar{K} = \sum_{\theta}\sum_{S} \frac{p(S|\theta)}{4} \sum_{n=\underline{\eta}}^{\overline{\eta}} \hat{K}_{n+1}(S,\theta). \tag{18}$$

5. Both labor markets, $S = 1, 2$, clear:

$$\bar{H}^S = \sum_{\theta} \frac{p(S|\theta)}{4} \sum_{n=\underline{\eta}+4(S-1)}^{\hat{n}(S,\theta)-1} e(n)\left(1 - \hat{I}_n(S,\theta)\right)\hat{H}_n(S,\theta). \tag{19}$$

6. The goods market clears:

$$\sum_{\theta}\sum_{S} \frac{p(S|\theta)}{4} \sum_{n=\underline{\eta}}^{\overline{\eta}} \hat{C}_n(S,\theta) \;+\; \sum_{\theta} \frac{p(2|\theta)}{4} \cdot 4\zeta \;+\; G + (1-\tau_k) r (1-\kappa)\bar{K}$$
$$= \tilde{F}(\bar{H}^1, \bar{H}^2, \bar{K}). \tag{20}$$

7. Government policy $\{\tau_l, \tau_k, \tau_p, \eta_p, P, G\}$ satisfies the government budget constraint:

$$G + (1-\tau_l)\sum_{\theta}\sum_{S} \frac{p(S|\theta)}{4}\Big(\overline{\eta} - \max\{\eta_p, \hat{n}(S,\theta)\} + 1\Big)P$$
$$= (\tau_l + \tau_p)(R^1\bar{H}^1 + R^2\bar{H}^2) + \tau_k r\bar{K}. \tag{21}$$

All parameterizations considered in this paper satisfy regularity requirement (i). Specifically, as explained in the last paragraph of subsection 2.2, since our parameterizations yield equilibrium after-tax interest rates that are greater than the subjective rate of discounting, and together with the other four listed features built into our model, front-loaded labor supplies are optimal. But as described in subsection 5.2, in some of our tax experiments agents with the same abilities and schoolings become indifferent between career strategies that differ in terms of retirement ages and human capitals accumulated on the job, regularity condition (ii) is not satisfied. In subsection 5.2, we show how to extend our definition of a stationary equilibrium to allow for such indifference by introducing equilibrium fractions of otherwise identical agents who nevertheless choose differing career strategies that yield the same lifetime utilities. Equilibrium fractions of agents who choose different strategies are pinned down in a stationary equilibrium.[12]

3 Calibration

To set the stage for our computational experiments, we parameterize our time-averaging model to mimic outcomes of HLT's analyses. First, we account for parameters that are either borrowed directly or "backed out" from HLT. Second, we make precise how our new parameters are calibrated to be compatible with HLT's parameter choices.

3.1 Parameters borrowed and "backed out" from HLT

HLT used both micro- and macroeonomic data to calibrate and estimate parameters. They wanted their model of human capital and earnings dynamics to match education and earnings outcomes of white male civilians using National Longitudinal Survey of Youth (NLSY) data for the period 1979–1993. To infer parameters of the aggregate production function they used aggregates from the National Income and Product accounts as well as data on workers in the Current Population Survey (CPS) for the period 1963–1993. Tables 1 and 2 summarize parameters inferred by HLT and that we have essentially imported into our time-averaging model.

Following a common calibration approach in applied macroeconomics, HLT (p. 15) posit "discount [$\delta = 0.96$] and intertemporal substitution [$\gamma = 0.9$] parameters in consumption to be consistent with those reported in the empirical literature and that enable us to reproduce key features of the macro data – like the capital–output ratio."[13] Besides a capital–output ratio of 4, HLT seek to target a steady state after-tax interest rate of $(1-\tau_k)r = 0.05$, to which we return below. HLT (pp. 26-27) assume a uniform tax rate on labor and capital income, $\tau_l = \tau_k = 0.15$, "that was suggested by Pechman (1987) as an accurate approximation to the true rate over our sample period once itemizations, deductions, and income-contingent benefits are factored in." Not reported by HLT, but gathered from one of the co-authors (Taber (2002, Table 1)), annual college tuition ζ is set equal to 1.02 (thousands of 1992 dollars).

Using data from the NLSY and measures of college tuition collected by the Department of Education, HLT estimated heterogeneities in human capital endowments, human capital

[12] Details on how to compute equilibria can be found in Appendix A.

[13] Be aware that we adhere to the formulation of preferences in (1) while HLT transforms the specification into one expressed in terms of $\hat{\gamma} \equiv 1 - \gamma = 0.1$.

Table 1: Same or similar parameterizations across HLT's and our models

Parameter	**HLT**	**Our** *	**Description**
δ	0.96	=	discount factor
γ	$0.9^{\dagger}$	1	intertemporal substitution
τ_l	0.15	=	labor income tax rate
τ_k	0.15	=	capital income tax rate
ζ	1.02	=	annual tuition in 4-year college
			Ben-Porath [high school / college]
α_1 / α_2	0.945 / 0.939	=	exponent on investment
β_1 / β_2	0.832 / 0.871	=	exponent on human capital
$A^1(1)$ / $A^2(1)$	0.081 / 0.081	=	productivity, ability $\theta = 1$
$A^1(2)$ / $A^2(2)$	0.085 / 0.082	=	$\theta = 2$
$A^1(3)$ / $A^2(3)$	0.087 / 0.082	=	$\theta = 3$
$A^1(4)$ / $A^2(4)$	0.086 / 0.084	=	$\theta = 4$
$H^1(1)$ / $H^2(1)$	8.042 / 11.117	=	initial human capital, $\theta = 1$
$H^1(2)$ / $H^2(2)$	10.063 / 12.271	=	$\theta = 2$
$H^1(3)$ / $H^2(3)$	11.127 / 12.960	=	$\theta = 3$
$H^1(4)$ / $H^2(4)$	10.361 / 15.095	=	$\theta = 4$
			Production function for goods
a_1	$0.496^{\ddagger}$	0.475	share high school (versus college)
a_2	$0.252^{\ddagger}$	0.235	share physical capital (versus human)
a_3	$2.504^{\ddagger}$	2.554	productivity
			substitution between
ρ_1	0.306	=	high school and college
ρ_2	-0.034	0	human and physical capital

* An equality sign means that our model adopts HLT's parameter value.

† See footnote 13.

‡ HLT report no parameter values for a_1, a_2, and a_3, which we instead deduce from our replication of HLT.

production technologies, and propensities to attend college. First, they constructed four equal-sized ability groups $\theta \in \{1, 2, 3, 4\}$ from scores on the Armed Forces Qualifying Test (AFQT) administered by the NLSY in 1980. Second, for each ability group θ and schooling choice S, they used data on earnings to estimate human capital endowment $H^S(\theta)$ and the parameters $A^S(\theta)$, α_S, and β_S in human capital technology (3). For any particular set of those parameters and under HLT's premise "that interest rates and the after-tax rental rates on human capital are fixed at constant but empirically concordant values," earnings profiles in the model can be deduced after computing optimal human capital investments over the lifecycle. Then by using nonlinear least squares estimation to minimize the discrepancy between those deduced earnings profiles and actual ones in the NLSY, HLT obtained the estimates of human capital parameters displayed in Table 1. Both $H^S(\theta)$ and $A^S(\theta)$ are mostly increasing in ability θ, except that high school graduates of ability group 3 have larger human capital endowment $H^1(3)$ and higher productivity $A^1(3)$ in the human capital technology when compared to ability group 4, differences consistent with earnings profiles for the two groups in the NLSY.

Third, HLT estimated an ability-specific stochastic process for the nonpecuniary cost ϵ of attending college. Under the assumptions of complete markets and inelastic labor supplies, utility-optimizing human capital investments maximize present values of lifetime after-tax labor earnings. HLT used maximization of the present value of after-tax labor earnings to characterize schooling decisions. Thus, an agent chooses to attend college if the present-value after-tax labor earnings as a college graduate, reduced by tuition and the nonpecuniary cost ϵ of attending college, exceeds the present value of after-tax income he would have earned as a high school graduate. The shock ϵ is expressed in dollars; its ability-specific means μ_θ are reported in Table 2, where negative numbers represent costs and positive numbers indicate a nonpecuniary benefit of attending college. For the first three ability groups, the mean cost decreases as ability rises, eventually turning into a mean benefit for ability group 3. However, this changes for the highest ability group 4 whose mean cost is the second largest, exceeded only by the mean cost of the lowest ability group 1. The common standard deviation σ of the ϵ-shocks is identified and estimated based on differences in college tuition across U.S. states. In the next section, in our time-averaging model, we will deduce counterparts to these nonpecuniary costs of attending college, but now expressed in terms of utils (as shown in expression (14)).

To infer parameters of the aggregate production function, HLT started with the usual decomposition of aggregate output into compensation to labor and capital based on the

Table 2: Nonpecuniary college cost $\epsilon \sim N(\mu_\theta, \sigma)$, expressed in thousands of dollars in HLT versus utils in our model*

Parameter	HLT	Our	Description
μ_1	-53.02	-6.819	ability-specific mean, $\theta = 1$
μ_2	-2.82	-2.808	$\theta = 2$
μ_3	29.77	-0.912	$\theta = 3$
μ_4	-28.65	-4.587	$\theta = 4$
σ	22.41	1.5	standard deviation

* A negative (positive) number represents a nonpecuniary cost (benefit) of attending college.

National Income and Product accounts. They inferred two human capital aggregates, high school human capital $\bar{H}^1$ and college human capital $\bar{H}^2$ together with corresponding skill prices, R^1 and R^2 from CPS data. They constructed the stock of physical capital $\bar{K}$ from Federal Reserve Board data sets. Except for the share parameters on the two human capital aggregates, they assumed that other parameters of production function (5) are time invariant; HLT specified a linear time trend for $\log[(1 - a_1)/a_1]$, and inferred a trend coefficient $\varphi = 0.036$. While such evidence on skill biased technological change informed estimates of parameters of production function (5), HLT did not use it to calibrate other parameters.[14]

When they set $\varphi = 0$, with one additional assumption the remaining parameters in Tables 1 and 2 generated HLT's initial steady before the mid 1970. To attain the calibration target of a capital-output ratio equal to 4, HLT (p. 27) imposed transfers from each retiring cohort to each new cohort that enters the labor market. Thus, "for each year, transfer X is taken from all workers at retirement age [65], and the total amount is equally distributed to all individuals (irrespective of ability) of age [18] in that period. For the simulations reported in this paper, $X \approx \$30,000$." As described in Section 2.1, our alternative approach for targeting the capital-output ratio is to assume that a share κ of the equilibrium capital stock is held by the agents in the model, a share to be calibrated in the next section.

[14] While acknowledging that the estimation of the human capital production technology "ignores the price variation induced by technological change," HLT (p. 19) point out that "a remarkable finding of our research, reported in [HLT's] Appendix B, is that this misspecification of the economic environment has only slight consequences for the estimation of the curvature parameters of the human capital technology."

In their main analysis, HLT (p. 28) "consider a permanent shift in technology toward skilled labor .... We start from an initial steady state [$\varphi = 0$] and suppose that the technology begins to manifest a skill bias in the mid 1970s [$\varphi = 0.036$] ... and continuing for 30 years" after which the economy converges to a new steady state. The shift in technology comes as a complete surprise to the agents. But once the technology change begins, agents know the entire future path of technology. Hence, from the mid 1970s HLT computed a perfect foresight equilibrium. In Table 3 we report some aspects of the initial and the future steady states.[15] In the next section, we use both steady states to calibrate remaining parameters in our time-averaging model to make it be compatible with HLT's parameter choices. We shall verify compatibility by assessing that simulations of our model mimic outcomes in HLT.

Because HLT reported neither a tuition cost ζ nor their estimates of production function parameters $\{a_1, a_2, a_3\}$, we do two more things in order to replicate HLT's findings. First, by targeting prices in HLT's baseline steady state, we can infer values of the three unreported production parameters. Second, after adding those three parameter values and a tuition cost from Taber (2002, Table 1) to the parameterization reported in HLT, we can reproduce outcomes in HLT's baseline steady state, as well as outcomes in HLT's second steady state in our subsequent simulation of their experiment of skill-biased technological change.

To provide more details about these specifications, the first two columns in Table 3 report our reconstructions of those two HLT steady states. Because the above calibration of the three production parameters targeted prices $\{r, R^1, R^2\}$ in HLT's baseline steady, these prices are replicated perfectly. In addition, our computed prices in the second steady state are very close to HLT's reported numbers $\{0.0609, 2.23, 2.41\}$ (see the text inside HLT's Figures 5 and 8). Likewise, our replicated end-of-life human capitals for different ability groups and schooling levels in the baseline steady state are very similar to those in HLT's Table I. A visual inspection of the leftmost points in HLT's Figure 9 shows that their "utilized" aggregates of human capital in the baseline steady state are close to our numbers $\bar{H}^1 = 274$ and $\bar{H}^2 = 280$. HLT do not report counterparts of the college enrollment rates that we report in our Table 3. Furthermore, our reproduction of life-cycle profiles for human capital investments and wages mimic those of HLT, as shown in Figure B.1 in Appendix B.[16]

[15] Regarding rental rates on human capital in the baseline steady state, HLT (p. 27) "calibrate the aggregate production parameters to yield ... rental rates on human capital of 2. These values are consistent with those used in estimation of the human capital production parameters. Since human capital is measured in terms of hourly wages, earnings from our simulations are annual income measured in thousand of dollars if agents

Table 3: Steady states in HLT and our model

Variable	**HLT*** Baseline	SBTC†	**Our model** Baseline	SBTC†
Retirement age, all workers	65	65	65	65
Interest rate, r	0.0588	0.0609	0.0588	0.0599
Rental rate on human capital				
high school, R^1	2	2.20	2	2.27
college, R^2	2	2.39	2	2.45
Inputs in goods production				
high school, $\bar{H}^1$	274	119	249	94
college, $\bar{H}^2$	280	446	287	459
physical capital, $\bar{K}$	5725	6814	5605	6849
College enrollment rates‡				
ability $\theta = 1$	0.09	0.38	0.11	0.47
$\theta = 2$	0.28	0.67	0.34	0.77
$\theta = 3$	0.56	0.89	0.56	0.90
$\theta = 4$	0.81	0.99	0.86	0.99
End-of-life human capital for high school / college workers				
ability $\theta = 1$	9.4 / 13.5	9.0 /12.9	9.2 / 13.0	9.0 / 12.8
$\theta = 2$	12.1 / 14.9	11.5 / 14.2	11.8 / 14.4	11.5 / 14.1
$\theta = 3$	13.6 / 15.5	12.9 / 14.8	13.2 / 15.0	12.8 / 14.7
$\theta = 4$	12.6 / 18.2	12.0 / 17.4	12.3 / 17.6	12.0 / 17.2

* Outcomes in HLT are based on our reconstruction of HLT.

† SBTC is the steady state after Skill-Biased Technological Change.

‡ College enrollment rates in the baseline steady state in HLT are based on our reconstruction of HLT, whereas the corresponding numbers in our model are those of Taber (2002, Table 1), to which our model is calibrated.

Table 4: New parameters in our model

Parameter	**Value**	**Description**
B	0.8	disutility of working
κ	0.388	fraction of capital held by agents
		Efficiency units of human capital
ϕ_1	0.2	slope coefficient
ϕ_2	75	age at inflection point
		Social security program
τ_p	0.1	payroll tax rate
η_p	65	age of eligibility for social security
P	8	social security benefit

3.2 New parameters compatible with HLT

Table 1 shows parameters from HLT that we have essentially imported into our model. Besides minor differences in production function parameters $\{a_1, a_2, a_3\}$ (to be explained below), the only other differences are that we "round off" $\gamma = 0.9$ to become 1 and $\rho_2 = -0.034$ to become zero, i.e., we adopt preference specification (2) with additively separable disutility of working, and a Cobb-Douglas production function (6). The latter formulations are commonly used in the macro literature; in the present context, these departures from HLT are immaterial. To calibrate our remaining parameters – the nonpecuniary costs of attending college in Table 2 and the new parameters in Table 4 – to be compatible with HLT's parameter choices, we deploy four steps.

As a first step, except for our having set $\gamma = 1$ and $\rho_2 = 0$, we assume that HLT's remaining parameterization in Table 1 also applies to our time-averaging model. Furthermore, we require that our equilibrium interest rate and skill prices must equal HLT's in Table 3. Subject to those restrictions, we explore alternative configurations of core features of our

work 2000 hours per year."

[16] An unresolved discrepancy between HLT and our replication is that our present-value earnings of high-school graduates of different abilities are between 6.9-7.0 percent lower than those reported in HLT's Table II, and for college graduates 7.1-7.2 percent lower. As these differences are similar across schooling levels, college enrollment rates are little affected if we calculate them using our present-value earnings or those of HLT.

time-averaging model – disutility of work at the extensive margin, diminishing efficiency units of human capital in old age, and a social security program – that induce all agents to retire at the official retirement age 65.

We posit a payroll tax rate $\tau_p = 0.10$ and target a social security benefit $P = 8$ that is equivalent to 40% of average earnings. This constant benefit implies a progressive replacement rate.[17] Parameters ϕ_1 and ϕ_2 in (4) that determine age-dependent efficiency units of human capital, in conjunction with the disutility B of working, can then reside in a nontrivial subspace that induces all agents to retire at age 65. For example, consider a degenerate case that lets ϕ_1 go to infinity and sets $\phi_2 = 65$ so that human capital produces efficiency units one-for-one until age 65, and then drops to zero. For sufficiently low values of B, all agents choose to retire at age 65, as in HLT's analyses. But because we want agents to be capable of working beyond age 65, we set the "slope" coefficient $\phi_1 = 0.2$ and move the inflection point to $\phi_2 = 75$ in (4). This generates a smoother decline in the efficiency units of human capital and the depreciation becomes noticeable only at ages in the 60s, as depicted by the solid line in Figure 1. Finally, given this suite of parameter values, we set the disutility of working $B = 0.8$ that lies in a mid-range of values for which all agents choose to retire at age 65. For a sensitivity analysis of these baseline parameters, see Appendix C.

In our second step, we calibrate probability distributions for nonpecuniary costs ϵ of attending college. We have altered HLT's specification by denominating that shock in utils instead of dollars. Consequently, we want to calibrate ability-specific means μ_θ and a common standard deviation σ to be compatible with these units. Ideally, we would want to use HLT's college enrollment rates as targets for our calibration, but HLT do not report those rates. So instead we again use findings of Taber (2002), the same source that gave us a value ζ of annual college tuition. For a given value of σ, we calibrate ability-specific means μ_θ to target college enrollment rates of different ability groups in Taber (2002, Table 1): we report these outcomes in our model's baseline steady state in Table 3. While postponing calibration of σ until our last step, our calibration of μ_θ lets us recover HLT's aggregate composition of high school and college graduates in the labor force, outcomes that will support skill prices close to those of HLT under our maintained assumption of an equilibrium interest rate equal to

[17] Relative to labor earnings at the time of retirement, the social security benefit $P = 8$ corresponds to a replacement rate of 52.2% (27.8%) for the lowest (highest) earner in our model, which reflects that the lowest earner earns about half of as much as the highest. In comparison, according to the OECD study of Queisser and Whitehouse (2006), the average gross replacement rate in the U.S. social security system for an individual with the average earnings is 38.6%, while it is 49.6% and 28.1% for individuals with half and twice the average earnings, respectively.

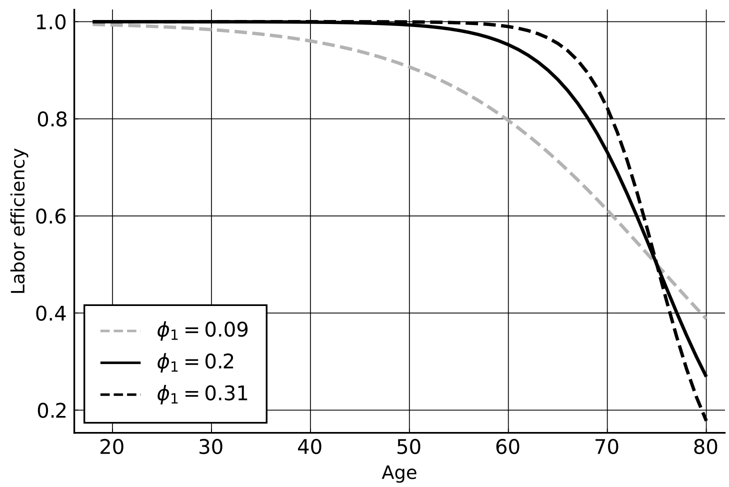

Figure 1: Age profile of the conversion of one unit of human capital into efficiency units.

HLT's.

Our third step ensures that our equilibrium interest rate agrees with HLT's. We back out a physical capital stock that supports that outcome and compute agents' net savings. The ratio of the latter to the former is our calibrated value of a new parameter κ, the fraction of physical capital held by agents who live inside our model; remaining capital is held by 'investors' who live outside our model. Since we are matching HLT's outcomes with a model that includes a pay-as-you-go social security program, we anticipate that our agents' private net savings fall far short of a physical capital stock required to generate the HLT's equilibrium interest rate. Not surprisingly, our calibrated value is $\kappa = 0.388$.

Our fourth step calibrates σ by using both steady states computed by HLT, namely, a baseline initial steady state to describe the early 1970s and a future steady state to which their economy converges after a 30-year period of skill-biased technological change, as described in Section 3.1. We calibrate σ by targeting the difference in the relative skill price ratio between the two steady states. HLT's permanent shift in technology toward skilled labor increases the price of college human capital relative to the price of high school human capital by 8%. Our calibrated value of σ comes from our finding of a positive relationship between that parameter and the implied increase in the relative skill price. A larger dispersion in the idiosyncratic nonpecuniary cost of attending college means that college attendance rates respond less to an increase in the relative skill price, so it takes a bigger change in skill prices to elicit reallocation from high school human capital to college human capital in response to skill-biased technological change.[18]

Tables 1, 2, and 4 list a complete parameterization of our time-averaging model. Besides the calibration issues discussed above, we make minor adjustments to HLT's production parameters $\{a_1, a_2, a_3\}$ in Table 1 in order exactly to target prices in HLT's baseline steady state. These minor adjustments are quantitatively insignificant in the sense that if we instead had adopted HLT's parameter values for $\{a_1, a_2, a_3\}$ in Table 1, the effects on our reproduction of HLT's outcomes in the last two columns of Table 3 would be unnoticeable, as would outcomes from the policy analyses below.

3.3 Model mimics outcomes in HLT

The last two columns of Table 3 show that our time-averaging model does a good job of approximating HLT's baseline steady state and also HLT's second steady state after skill-biased technological change.[19] To understand sources of these close approximations, we describe how primitives inherited from the HLT model interact with key new features of endogenous career length and a social security system that we have put into our time-averaging model.

In contrast to HLT's assumption of inelastic labor supply until exogenous retirement at age 65, career length and time of retirement are endogenous in our model. As in HLT, after agents draw their abilities and their nonpecuniary costs of attending college, they face no risks, so a competitive market in risk-free one-period loans is enough to ensure efficient on-the-job human capital investments *conditional* on a choice of career length. For a given career length, optimal on-the-job human capital investments are those that maximize the present value of labor earnings. Hence, those investment decisions are decoupled from a worker's choice of when to consume goods. But the choice of career length is *not* decoupled in that way because of how it determines utility derived from leisure in retirement. Furthermore, since the after-tax market interest rate is higher than the subjective discount rate in both

[18] Computationally, our fourth step of calibrating σ involves a loop over the second and third step. The loop is initiated with a guess of σ in the second step and the candidate parameterization of the baseline model that emerges after the third step is used to obtain the future steady state after skill-biased technological change. If the implied increase in the relative skill price in that second steady is greater (smaller) than 8%, we revise our guess of σ downward (upward) and restart another loop over the second and third step.

[19] In the baseline steady state, our life-cycle profiles for human capital investments and wages are also very close to those of HLT, as can be seen when comparing Figure B.2 to Figure B.1 in Appendix B. The only difference is that the levels of savings are significantly lower in our model as compared to HLT. The reason is that we have introduced a pay-as-you-go social security system in our model, as elaborated upon at the end of Section 2.1.

HLT's analysis and ours, an agent prefers to front-load lifetime labor supply in order to earn a return on savings accumulated from past wages that exceeds his time preference discount rate. All of these considerations affecting endogenous labor supplies become moot in our baseline steady state because the social security system induces agents to put themselves on a corner that involves working until the official retirement age 65. As a consequence, all workers in our baseline steady state find it optimal to maximize their lifetime after-tax labor incomes before retiring at age 65, just as in HLT's model. Since we adopt the same human capital technologies as HLT, it is not surprising that human capital investments and lifetime earnings profiles in our baseline steady state closely approximate HLT's.

There is no presumption that our time-averaging model can also do a good job of approximating HLT's second post-skill-biased technological change steady state. Thus, we might anticipate that in our model some agents might leave the corner solution that tells them to retire at the official retirement age 65. Other differences might arise because other auxiliary assumptions that influence the aggregate stock of physical capital. Thus, recall HLT's constant per capita lump-sum transfer X from the old to the young that they adjust to target their baseline steady-state capital-output ratio; we instead hit that target by assuming that agents who live inside our model hold only a fraction κ of the equilibrium capital stock and are entitled to social security benefits at age 65 if they have chosen to retire then. Despite these important difference in primitives, we find that our time-averaging model does a good job of approximating HLT's post-technical-change steady state. A critical outcome in our model is that agents continue to retire at the official retirement age 65. So once again, with the same human capital technologies and now also having experienced the same technological change as the agents in HLT's model, agents in our model enroll in college and invest in human capital at the same rates as do the agents in HLT's model.

Thus, HLT could have used our time-averaging model with endogenous retirement to get their same quantitative findings. Our model with a social security system provides choice-theoretic rationalizations for retirement outcomes that HLT hard-wired.[20]

Although these structural differences between our model and HLT's make no practical differences for the issues studied by HLT, they do matter when we study social security reforms and possible employment effects of increasing the labor income tax studied by Prescott (2002).

[20]A common practice is to justify exogenously requiring agents to retire at age 65 by alluding informally to rules of a social security system that used to prevail before recent reforms designed not to discourage working beyond age 65.

4 Social security reform

As our first policy experiment we assume that all workers receive social security benefits from age 65, regardless of when they choose to retire. After beginning-of-life abilities and nonpecuniary costs of attending college are realized, agents face no risks and they borrow and lend at a risk-free interest rate, so the only consequence of the social security reform is to remove an implicit tax on working after the official retirement age.

The first two columns in Table 5 show retirement ages under this reform when prices are kept fixed and in a general equilibrium, respectively, where age i (j) in entry 'i/j' is the retirement age of a high school (college) worker. In the first column, both the interest rate and skill prices are kept constant at their values in the baseline economy; in addition, we also froze the ability composition when we computed an average retirement age. Since at fixed prices before the reform they had chosen a corner solution at the official retirement age 65, all workers choose to extend their career lengths. While high school workers increase their career lengths on average by 2.4 years, college educated workers increase their average retirement age of by 7.6 years to 72.6. These changes are attenuated in a general equilibrium: high school workers actually choose to retire *early*, on average one year before the official retirement age. This experiment thus unleashes countervailing forces that we shall unbundle by successively perturbing salient aspects of the environment.

Three forces induce high school workers to retire earlier than college workers. First, the social security system redistributes from high-ability to low-ability agents. Since all workers pay the same proportional payroll tax on labor incomes, the equal social security benefit s awarded to all retirees mean that agents with low earnings receive more than they pay into the system. Income effects of that net transfer to low-ability agents, who are more likely to be high school workers, induces them to supply less labor. Our first perturbation is designed to quantify this effect: we simply remove the social security system so that younger agents must save in order to consume during their retirements. Except for recalibrating the fraction of the economy's capital stock that is held by agents who live inside our model,[21] we retain

[21] As discussed in Section 3.2, the pay-as-you-go social security system in the baseline economy significantly suppresses agents' private retirement savings. Hence, the removal of social security in our perturbed model increases those savings. For reasons of comparability, we therefore recalibrate the fraction of the capital stock held by the agents in the model to preserve HLT's calibration target of a capital-output ratio equal to 4. Doing that calls for an increase in the fraction of the capital stock held by the agents in the model from 0.388 to 0.767. We note that with that recalibration we can replicate both the baseline and the second steady state of HLT in our perturbed model, conditional on adding the restriction that agents cannot work beyond age 65.

Table 5: Retirement ages [high school/college workers] with and without social security

Ability	**Social security reform**		**No social security**	
	fixed prices *	general eq.	new baseline †	perturbed techn. ‡
$\theta = 1$	66.7 / 72.8	63.3 / 70.9	64.0 / 71.0	66.1 / 70.3
$\theta = 2$	67.7 / 72.8	64.1 / 70.9	64.6 / 71.0	66.0 / 70.3
$\theta = 3$	68.2 / 72.5	64.6 / 70.6	65.1 / 70.7	65.5 / 70.1
$\theta = 4$	68.2 / 72.7	64.5 / 70.8	65.0 / 70.8	65.8 / 70.2
Average	67.4 / 72.6	63.9 / 70.8	64.5 / 70.8	65.9 / 70.2

* Both the interest rate and skill prices are kept constant at the values of the baseline economy, as well as the ability composition when computing an average retirement age.

† In the new baseline of an economy without social security, the fraction of the capital stock held by the agents in the model are recalibrated so as to maintain a capital-output ratio equal to 4.

‡ Human capital technologies are perturbed so that high school and college workers of the same ability have the same technology and initial human capital, equal to those of the latter workers.

our parameterization of the baseline economy and compute a steady state without social security. The third column of Table 5 shows that workers who had chosen to retire earliest under the social security reform – high school workers of the lowest ability – increase their career lengths marginally more than other workers in response to removing all social security benefits.

Second, recall Ljungqvist and Sargent's (2014) finding that the more elastic is an earnings profile to accumulated time worked, the longer is a worker's optimal career. The same force is present in our model here with its Ben-Porath human capital technology. To quantify this force, our next perturbation of the economy without social security endows high school workers with the same human capital accumulation technology that college workers have. This technology provides higher returns to time devoted to human capital accumulation on the job than does the one that high school workers ordinarily have in our model. Thus, our perturbed human capital technology parameters, denoted $\{\hat{\alpha}_S, \hat{\beta}_S, \hat{A}^S(\theta), \hat{H}^S(\theta)\}$, satisfy $\hat{\alpha}^1 = \hat{\alpha}^2 = \alpha^2$, $\hat{\beta}^1 = \hat{\beta}^2 = \beta^2$; and, for each θ, $\hat{A}^1(\theta) = \hat{A}^2(\theta) = A^2(\theta)$, and $\hat{H}^1(\theta) = \hat{H}^2(\theta) = H^2(\theta)$. We keep all other parameters the same as in the preceding perturbed economy without social security. As the fourth column of Table 5 shows, letting high school workers have college workers' human capital technology induces them to choose longer careers in the new steady state.

Third, we ascribe to yet another effect of time averaging the approximately 4-year difference in career lengths between high school and college workers in the fourth column of Table 5. If to the bare-bones time-averaging framework of Ljungqvist and Sargent (2006, 2014), we were to add the assumption that an initial apprenticeship period Z of a labor market career yields no labor earnings but is required before starting gainful employment, at an interior solution an optimal career length is the sum of Z and a corresponding optimal career length in an economy without that apprenticeship.[22] This force is at work in our present time-averaging version of an HLT framework. Thus, in the perturbed model without social security and identical human capital accumulation technologies for high school and college workers of the same abilities, if an agent finds it optimal to acquire a college degree, he apparently treats the years in college as a fixed requirement and simply tacks them on to the length of time spent working that he would have chosen if he had instead gone to work straight out of high school: regardless of schooling choice, the optimal time spent actually working depends only on the fixed disutility of working and the human capital accumulation technology that influences how agents choose to shape their earnings profiles. Furthermore, because workers' preferences are compatible with balanced growth, absolute wage levels *per se* do not affect labor supply decisions, so income and substitution effects cancel.[23]

[22]Consider the simplest version of the time averaging model in continuous time studied by Ljungqvist and Sargent (2006, 2014). Over a deterministic lifespan of unit length, an agent maximizes lifetime utility $\int_0^1 e^{-\varrho t}[\log(c_t) - Bn_t]\,dt$ subject to a present value budget constraint $\int_0^1 e^{-rt}[wn_t - c_t]\,dt \geq 0$. Under the assumptions that the agent's subjective discount rate ϱ and the market interest rate r are the same and equal to 0, the optimization problem reduces to

$$\max_T\,[\log(c) - BT] \quad \text{subject to } \; c = wT, \;\; c \geq 0, \;\; T \in [0,1]\,,$$

i.e., the agent chooses a constant consumption stream c and a fraction T of the lifespan devoted to working. At an interior solution, the optimal career length is $T = 1/B$. Perturb the model by assuming that an initial interval $Z \in [0, 1)$ of a career yields no labor income so that Z is time spent to acquire necessary training. The optimization problem then becomes

$$\max_T\,[\log(c) - BT] \quad \text{subject to } \; c = w \cdot \max\{T - Z, 0\}, \;\; c \geq 0, \;\; T \in [0,1]\,.$$

At an interior solution, the optimal career length is $T = 1/B + Z$.

[23]In Appendix D we consider other social security reforms that retain our baseline assumption that receiving benefits is conditional on retirement but that increase social security benefits. As benefits are raised above the baseline level, college educated workers continue to retire at 65, but high-school educated workers retire earlier and earlier. This is consistent with our findings in Table 5. Both the reform summarized in

5 Taxation and spending

To study responses of labor supplies to increases in labor income tax rates that depend on how the government allocates tax revenues, we conduct a tax experiment along lines of Prescott (2002). Prescott argued that most tax revenues are spent for goods and services that substitute perfectly for private consumption, so he modeled government expenditures as lump-sum transfers to households. Temporarily embracing Prescott's assumption, we explore effects of increasing our labor income tax rate from its baseline parameterization of $\tau_l = 0.15$. At higher tax rates $\tau_l > 0.15$, a fraction $(\tau_l - 0.15)/\tau_l$ of all tax revenues raised by levying the tax rate τ_l on labor income is returned to each agent as a lump-sum transfer. Two remarks are pertinent. First, we note that the lump-sum transfer includes no tax revenues from levying the tax rate τ_l on social security benefits. Second, because the baseline tax rate is 15 percent, revenues from the higher tax rate τ_l are in general higher than the change in total revenues from labor income taxation. Smaller tax revenues can be expected to be raised from the 15-percent baseline portion of the labor income tax whenever a higher tax rate τ_l causes labor supplies and consequently aggregate labor income to shrink. Such losses of baseline tax revenues do not affect our calculation of a lump-sum transfer. Our tax experiment risks bankrupting the government, so as we raise the labor income tax rate, we must verify that the sum of labor income tax revenues not handed back as lump-sum transfers and the revenues from the capital tax and the payroll tax are sufficient for the government to finance social security benefits.

5.1 Aggregate outcomes

The solid line in Figure 2(a) depicts a Laffer curve in the tax rate when tax revenues are handed back as lump-sum transfers. Since the tax revenues are expressed in per capita terms, the solid line is also the lump-sum transfer to each agent at different tax rates $\tau_l \geq 0.15$. The Laffer curve peaks at labor tax rate $\tau_l = 0.54$. By way of contrast, the dashed line in Figure 2(a) depicts the Laffer curve that would prevail if tax revenues were instead to be used to finance government expenditures that are *not* good substitutes for private consumption. The two Laffer curves ratify Prescott's (2002, p. 7) assertion that "the assumption that the tax revenues are given back to households either as transfers or as goods and services [that

that table and the reform analyzed in Appendix D indicate that it is college-educated workers who are most affected by the kink in budget constraints implied by the implicit tax contained in the baseline social security policy: they would retire significantly later if such a kink were to be removed and would not choose to retire earlier even if benefits were significantly increased under the baseline system.

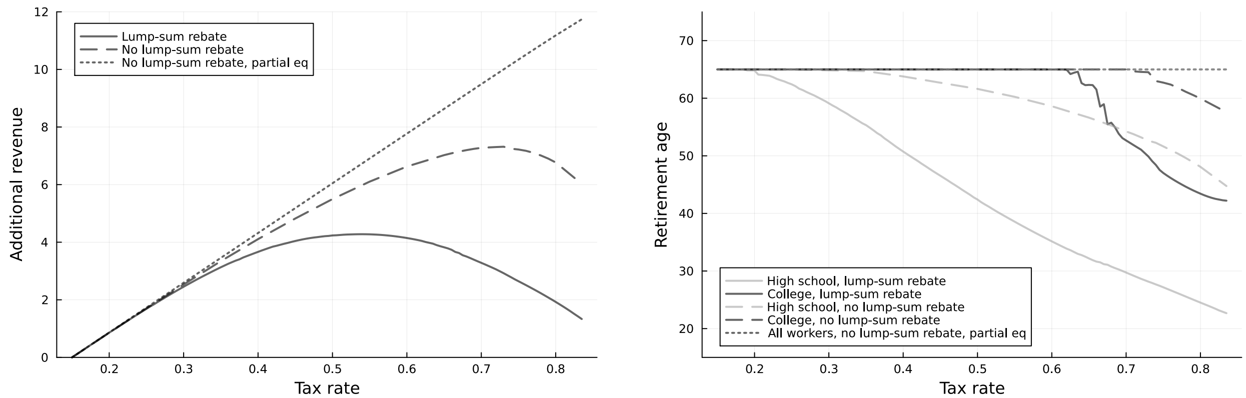

Figure 2: Laffer curves and retirement ages as functions of the labor tax rate that depend on whether or not tax revenues from labor taxation above the baseline rate $\tau_l = 0.15$ are handed back as equal lump-sum transfers to agents. In Panel A, the solid and dashed lines are Laffer curves with and without lump-sum rebates, respectively. In Panel B, the dark and light solid (dashed) lines are the average retirement ages of college and high school workers, respectively, with (without) lump-sum rebates. In both panels, dotted lines depict partial-equilibrium outcomes in the economy without lump-sum rebates, when the interest rate is kept constant at the baseline equilibrium rate. Notice how the single dotted line in Panel B indicates that all workers retire at age 65 throughout the computed tax range.

are good substitutes to private expenditures] matters. If these revenues are used for some public good or are squandered, private consumption will fall, and the tax wedge will have little consequence for labor supply." Nevertheless, because capital formation is affected in a general equilibrium, the tax wedge on labor income brings distortions that increase along with the tax rate. Thus, the dashed Laffer curve peaks at tax rate $\tau_l = 0.73$ for the economy without lump-sum rebates. To examine how much of the distortions operate through capital formation, the dotted line in Figure 2(a) is the Laffer curve for a small-open version without lump-sum rebates when the interest rate is held constant at the baseline equilibrium rate.

Effects of tax distortions on labor supplies manifest themselves as changes in career lengths, college enrollment rates, and how workers allocate their time between working and accumulating human capital on the job. Figure 2(b) depicts effects of the labor income tax rate on retirement ages. Dark and light solid lines show average retirement ages of college and high school workers, respectively, in the economy with lump-sum rebates. At the baseline tax rate $\tau_l = 0.15$, outcomes are those of the baseline steady state of Section 3.3. High school workers are the first to start retiring early in response to higher tax rates and associated lump-sum transfers, while college workers retire early only when tax rates are higher than $\tau_l = 0.60$. Counterparts in the form of dashed lines show that these effects

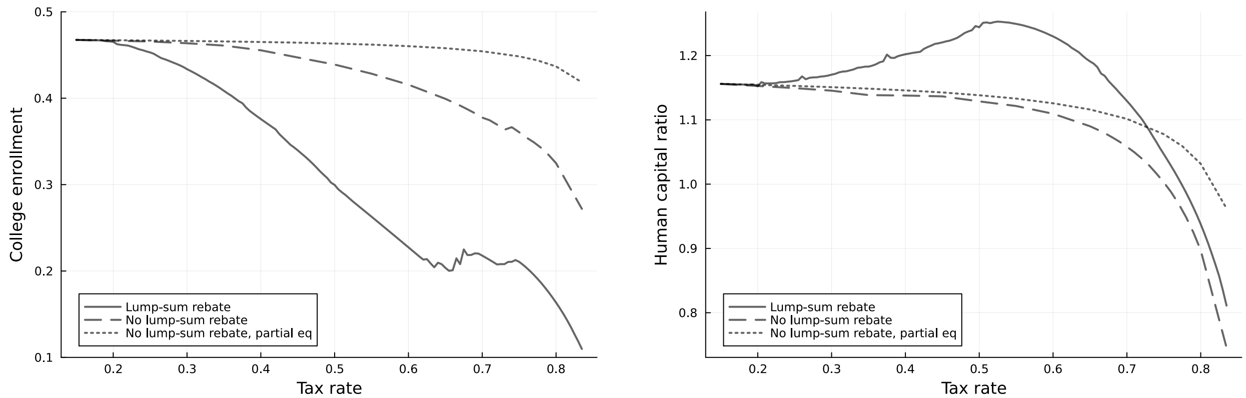

Figure 3: College enrollment (Panel A) and ratio of college to high school human capital in the production of goods (Panel B) as functions of the labor tax rate depend on whether or not tax revenues from labor taxation above the baseline rate $\tau_l = 0.15$ are handed back as equal lump-sum transfers to agents. In both panels, solid and dashed lines display outcomes with and without lump-sum rebates, respectively. The dotted lines depict partial-equilibrium outcomes in the economy without lump-sum rebates when the interest rate is kept constant at the baseline equilibrium rate.

on retirement ages are attenuated in an economy without lump-sum rebates. Furthermore, in the small-open version of our no-lump-sum-rebate economy, over the computed tax range all workers continue to work until age 65, i.e., the dotted line in Figure 2(b).

In the economy with lump-sum rebates, shortenings of high school workers' career lengths and labor supplies of college workers that are less sensitive to tax rate increases are reconciled in a general equilibrium through a falling college enrollment rate, as indicated by the solid line in Figure 3(a). The depressed labor supply of the average high school worker brings adjustments of equilibrium prices that induce more agents to commence working with high school degrees instead of going to college. Workers' choices of how much human capital to accumulate on their jobs also influence aggregate quantities of college and high school human capitals employed to produce goods. Connections between these choices and workers' decisions about when to retire result in the almost flat segment in college enrollment at the upper range of taxes, to be discussed in Section 5.2. Taken together, as shown by the solid line in Figure 3(b), these forces on labor supplies result in a ratio of aggregate college to high school human capital devoted to producing goods that initially increases with the tax rate, peaks around the same tax rate as does the Laffer curve in Figure 2(a), and then starts falling ever more rapidly, eventually in tandem with a sharp decline in the college enrollment rate in Figure 3(a).

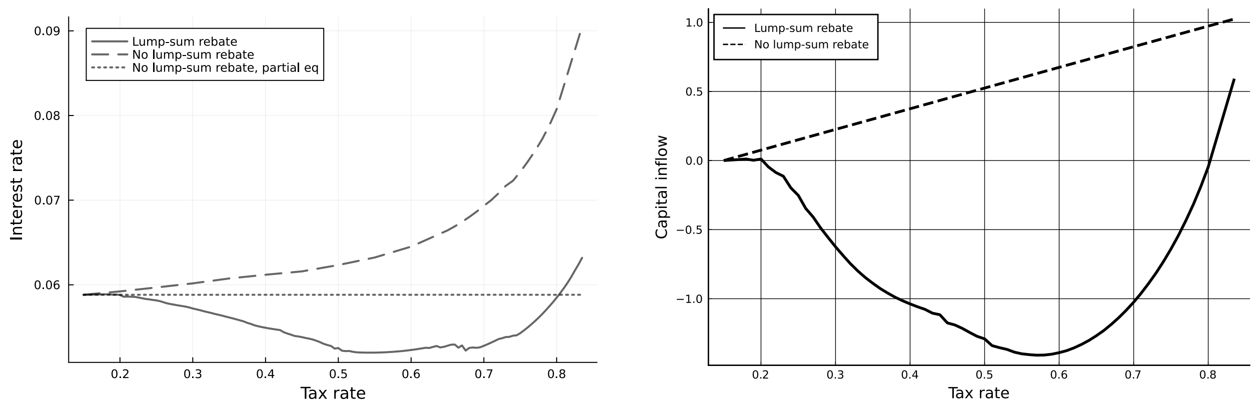

Figure 4: Interest rate (Panel A) and capital inflow into small open economy (Panel B) as functions of the labor tax rate and whether or not tax revenues from labor taxation above the baseline rate $\tau_l = 0.15$ are handed back as equal lump-sum transfers to agents. In both panels, the solid and the dashed lines display outcomes with and without lump-sum rebates, respectively. The dotted line in Panel A depicts the constant interest rate in a small open economy, i.e., our baseline equilibrium interest rate.

The dashed line in Figure 3(a) shows that effects on college enrollment are attenuated in the economy without lump-sum rebates and, according to the dotted line, are even more attenuated in a small-open economy version. In the absence of lump-sum rebates, the ratio of college to high school human capitals used to produce goods decreases monotonically as the tax rate rises. To gain a better understanding of these outcomes, we inspect the equilibrium interest rate and prices of the two types of human capital.

In the small-open version of the economy without lump-sum rebates, the interest rate is kept constant at the baseline equilibrium rate 0.059 ($= 0.05/(1 - \tau_k)$), as indicated by the dotted line in Figure 4(a). According to the dashed line in Figure 4(b), increases in the labor tax rate are accompanied by a capital inflow from abroad, expressed as a fraction of the equilibrium capital stock at each tax rate. Here is what happens. Consider a standard laissez-faire growth model with preferences consistent with balanced growth, as is true for our utility function (2). A permanent decline in a multiplicative productivity parameter would cause a proportional reduction in the wage rate that would leave steady-state labor supply unchanged as substitution and income effects cancel. Other things equal, a similar cancellation of income and substitution effects would occur if the take-home wage rate were instead to be reduced by levying a proportional labor income tax (with all tax revenues being squandered) and hence, steady-state labor supply would remain unchanged. But capital formation rates are not equal across the two settings. While a multiplicative deterioration of the production function would leave all relevant equilibrium ratios unchanged including the fraction of agents' income devoted to investments that sustain the new steady-state capital stock, such invariance of equilibrium ratios would not prevail if the reduction of agents' take-home wage rate came about because of a proportional labor tax. In particular, since the production technology has not changed, the capital stock would need to stay unchanged in order to justify our temporary assumption of an unchanged before-tax wage rate upon which the cancellation of substitution and income effects hinges. But the capital stock would have to change because the investments required to sustain that unchanged capital stock constitute a larger share of agents' now depressed after-tax income. However, if we were to assume a small-open economy with an interest rate held constant at the economy's steady-state rate prior to the imposition of the proportional labor tax, outcomes would indeed be the same as if there had been a multiplicative deterioration of the production function. In the words of Prescott (2002, p. 7) above, "private consumption will fall, and the tax wedge will have little consequence for labor supply." Indeed, there would be no effect at all on labor supply because the capital flowing into the economy from abroad would completely make up for the shortfall in domestic savings at the unchanged interest rate. This reasoning explains the capital inflow along the dashed line in Figure 4(b) that sustains the constant interest rate indicated by the dotted line in Figure 4(a), which in turn rationalizes the nearly linear dotted Laffer curve in Figure 2(a) that implies an approximately unchanged labor supply over the depicted tax range.[24]

This reasoning also explains the monotonically increasing interest rate along the dashed line in Figure 4(a) for our economy without lump-sum rebates. Thus, to offset the capital inflow that occurs in the small-open economy version, the general-equilibrium interest rate has to rise in order to increase agents' savings and to reduce firms' demand for capital services. While our assumption of preferences consistent with balanced growth props up labor supplies in response to a proportional labor tax when tax revenues are squandered or spent on public goods that are not good substitutes for private expenditures, agents' propensities to save enough to maintain the capital stock are now suppressed by their diminished after-tax incomes. So the interest rate must rise.

The picture gets more complicated if tax revenues are returned lump-sum. Lump-sum rebates suppress the income effect of the proportional labor tax and give substitution effect

[24]In contrast to our account of outcomes in a standard growth model, only an approximate invariance of labor supply to a proportional labor income tax prevails in the small-open economy version of our model. This comes from our added feature of a schooling choice with a nonpecuniary cost of attending college, as will be explained below.

free rein to reduce labor supply. As for the equilibrium interest rate, we make two observations. First, in the lump-sum-rebate economy, countervailing forces no longer operate in the capital market. Instead, since the labor tax rate now has a strong suppressive effect on labor supply, the lower savings of workers with their smaller incomes and the lower demand for capital services by firms producing less ouput go hand in hand as the tax rate increases. Consequently, we can no longer argue that the interest rate must unequivocally rise in response to a higher tax rate as it does in the no-lump-sum-rebate economy. Second, lifecycle savings forces lead us to anticipate that the interest rate might now fall in response to a higher tax rate. This is because, while shortened career lengths reduce agents' labor income, it also increases their motive to save parts of that labor income early in life to prepare for more years of not working when retired. This exerts a downward pressure on the interest rate. However, this force should dissipate at high enough tax rates when the lump-sum transfer becomes large relative to net-of-tax lifetime labor earnings. The transfer effectively shifts agents' disposable incomes forward over their lifetimes, supplementing their net-of-tax social security benefits in old age. The resulting decline in agents' demand for private savings to finance future consumption exerts an upward pressure on the interest rate in response to any further increases in the tax rate. These forces play out in Figure 4(a) where the solid line shows that the interest rate is a U-shaped function of the tax rate. The corresponding outcome for capital flows in the small-open version of the lump-sum-rebate economy is depicted by the solid line in Figure 4(b) .

Figure 5 depicts the relative price of college and high school human capitals – the skill premium; qualitatively it resembles Figure 4(a) that shows the interest rate. In the economy without lump-sum rebates (dashed lines), increases in both the skill premium and the interest rate as the labor tax rate rises emerge from countervailing forces affecting the attractiveness of attending college. A higher skill premium increases returns to a college degree, but a higher interest rate lowers it because costs in the form of tuition and four years of lost labor earnings are incurred upfront and future higher earnings as a college graduate are subject to a higher discount rate. These countervailing forces come from how the interest rate affects investments in both physical and human capital and how a general equilibrium adjusts prices to reconcile outcomes. Our focus above was on how movements in the interest rate can offset what otherwise would be capital flows in a small-open version of the economy. As a force that moderates impacts of the interest rate on human capital investments, movements in the skill premium create incentives to attend college. To close the general-equilibrium circle, prices of human capital are equated to marginal products of college and high school human

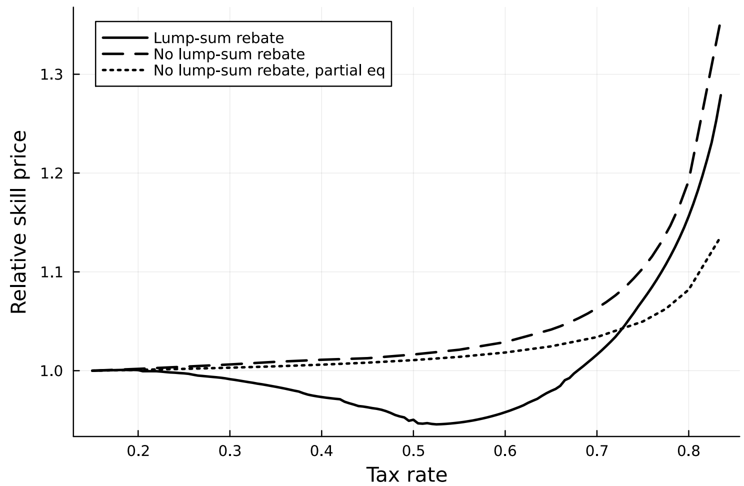

Figure 5: Relative price of college to high school human capital as a function of the labor tax rate and whether or not tax revenues from labor taxation above the baseline rate $\tau_l = 0.15$ are handed back as equal lump-sum transfers. The solid and the dashed line displays outcomes with and without lump-sum rebates, respectively. The dotted line depicts partial-equilibrium outcomes in the economy without lump-sum rebates when the interest rate is kept constant at the baseline equilibrium rate.

capitals, so the relative price in Figure 5 should indeed be an inverse image of the ratio of those two inputs in production that is depicted in Figure 3(b).

Reverse forces are evidently present in the economy with lump-sum rebates (solid lines) as both the interest rate and the skill premium at first decrease with increases in the labor tax rate in Figures 4(a) and 5, respectively. A falling interest offsets what otherwise would provoke a capital outflow in the small-open economy version. Evidently, this falling interest rate stimulates investments in college human capital so much that the skill premium falls. For high enough tax rates, both the skill premium and the interest rate will increase in the economy with lump-sum rebates, as they do in the economy without lump-sum rebates. While the same prime force increases the interest rate in these two economies, paths of causation differ. The prime force is that agents in both economies are deprived of resources to invest in physical capital. This is obvious for the economy without lump-sum rebates since agents are immediately left with lower after-tax incomes. In the economy with lump-sum rebates agents also eventually end up with reduced resources to invest in physical capital, but now this happens because the tax-transfer policy is so powerful in disincentivizing labor supplies that the economy's output plummets, and the transfers reshuffle agents' disposable incomes over time so that little is available to be invested in physical capital, as described above. These different paths of casuation manifest themselves in the Laffer curves in Figure 2(a). The solid Laffer curve for the economy with lump-sum rebates approaches zero, reflecting vanishing aggregate production. In contrast, the dashed Laffer curve for the economy without lump-sum rebates is higher up with plenty of tax revenues for the government to squander or to finance public goods that are not close substitutes to private expenditures, while agents are deprived of resources for private consumption and investments in the capital stock.

While the interest rate cannot change in the small-open version of the economy without lump-sum rebates, notice an eventual increase in the relative price of college to high school human capital (dotted line) in Figure 5. The skill premium compensates a marginal agent for investing in a college degree. Besides payments of tuition, as the idiosyncratic nonpecuniary cost ϵ of attending college becomes larger relative to the diminished lifetime utility of consumption when agents are deprived of resources, the skill premium must increase to compensate agents who choose to become college graduates instead of high school workers.[25]

We turn next to individuals' decisions that shape these aggregate outcomes.

5.2 Individual outcomes

In this section, we focus on agents' behavior and outcomes in the economy with lump-sum rebates: tax revenues from labor taxation above the baseline rate $\tau_l = 0.15$ are returned as lump-sum transfers to all agents.

As in HLT's model, heterogeneity in our baseline equilibrium occurs in the form of 8 distinct classes indexed by 8 pairs consisting of an agent's endowed ability, which belongs to one of four possible ability groups, and an agent's schooling level, which can be either a high school or a college graduate. At high enough tax rates, it is possible that some agents with the same ability and schooling are indifferent between career strategies that differ in terms of retirement age and human capital accumulated on the job. We describe this endogenous type of heterogeneity below.

The data indicate a high correlation between levels of ability as measured by test scores and an individual's propensity to acquire a college degree. Differences in rates of college enrollment across abilities in our baseline steady state, as reported in the third column of

[25]While the present section has reported on outcomes in a small-open version of the economy without lump-sum rebates, Appendix E provides a corresponding analysis of a small-open version of the economy with lump-sum rebates.

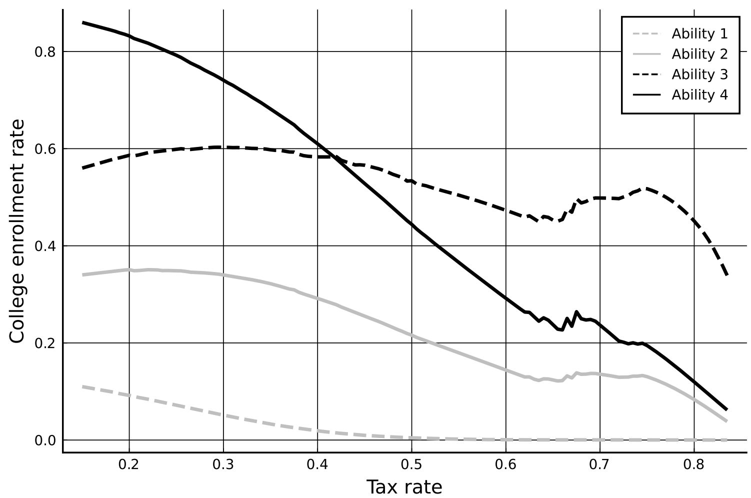

Figure 6: College enrollment rates by ability groups as functions of the labor tax rate.

Table 3, appear in Figure 6 at baseline labor tax rate $\tau_l = 0.15$. Those college enrollment rates respond to changes in the tax rate in ways that depend on our having embraced HLT's estimates of ability-specific distributions of nonpecuniary costs of attending college and ability-specific initial endowments of high school and college human capital. To match evidence of high earnings and steep earnings profiles of college workers of the highest ability group 4, HLT inferred that their initial endowment of college human capital is high and that they also reap a high return from additional investments in human capital on the job. To explain how a non-negligible fraction of agents in ability group 4 nevertheless chooses to supply labor as high school workers who actually earn less than high school workers from the next highest ability group 3, HLT imputed to them a high average disutility of attending college. Thus, despite large relative and absolute advantages of becoming college workers, their high disutilities of attending college cause 14 percent of agents in ability group 4 to become high school workers in the baseline steady state. Those high disutilities of attending college explain why the college enrollment rate of ability group 4 falls most sharply in response to higher labor income taxation in Figure 6. At the opposite end of the ability spectrum, only ability group 1 has a higher average disutility of attending college than ability group 4. Ability group 1 also has by far the lowest endowment of college human capital. Consequently, only 11 percent of agents in ability group 1 attend college in the baseline steady state; that fraction falls to less than one percent at labor tax rates above 0.45 in Figure 6.

In addition to depending on college enrollment rates, supplies of high school and college

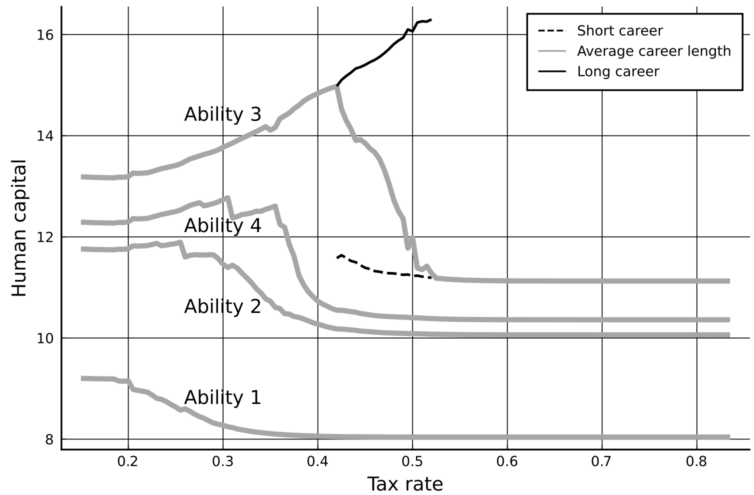

Figure 7: End-of-life human capital stocks of high school workers by ability group as functions of the labor tax rate. As for ability group 3, over the tax range 0.42–0.52, workers are indifferent between accumulating relatively high and low stocks of human capital, as depicted by the dark solid and dark dashed line, respectively, whereas the light solid intermediate line shows a worker's average human capital in an equilibrium.

human capital also depend on how much human capital agents accumulate on the job. Figure 7 shows end-of-life human capitals of high school workers of the four ability groups as functions of the labor tax rate. These are measured in raw units of human capital, not age-dependent efficiency units; given the absence of depreciation from (3), an agent's end-of-life human capital is the maximal human capital attained over his labor market career. For the lower ability groups 1 and 2, human capital accumulation tends to decrease as the tax rate increases – an adverse effect of taxation that could have been anticipated. Although that outcome also eventually prevails for ability groups 3 and 4, initially we see opposite outcomes whenever end-of-life human capital increases with increases in the tax rate in Figure 7. This pattern reflects general equilibrium effects when the falling supply of high school human capital of workers from the lower ability groups provokes higher supplies from the two higher ability groups. Initially, along the solid line in Figure 4(a), a falling interest rate motivates those higher ability workers to accumulate more human capital. In contrast, a high school worker from the lowest ability group 1 monotonically reduces human capital in response to a higher tax rate and likewise, a worker from the second lowest ability group 2 is prone to begin to reduce human capital accumulation. Equilibrium lump-sum transfers of tax revenues form larger shares of prospective labor incomes of low-income workers, so that

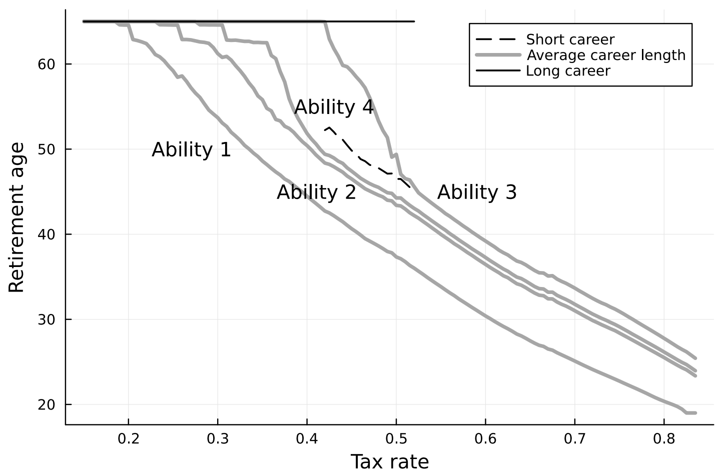

Figure 8: Retirement ages of high school workers by ability groups as functions of the tax rate on labor earnings. Farthest to the left, all workers retire at the official retirement age 65. Next, for ability groups 1,2 and 4, retirement ages decline gradually at higher tax rates. Regarding ability group 3, over the tax range 0.42–0.52, workers are indifferent between retiring at age 65 on the dark solid line and at an earlier age on the dashed line: the light solid intermediate line shows the average retirement age in an equilibrium. Thus, when the light solid line merges with the dashed line, all workers of ability group 3 retire early.

the tax-and-transfer system more adversely affects labor supplies of lower-income workers.

When high school workers accumulate less human capital in Figure 7, they also shorten their career lengths as shown in Figure 8. Except for those from ability group 3, this shortening of career lengths in response to higher labor income taxation happens gradually for high school workers. In equilibria over the tax range 0.42–0.52, high school workers of ability group 3 are indifferent between two starkly different career strategies: working until the official retirement age 65 with high end-of-life human capital versus retiring much earlier with little human capital. To shed light on these outcomes, Figure 9 shows lifetime utilities of a high school worker, conditional on working continuously until various retirement ages, while choosing consumption and accumulating human capital optimally. For each ability group, a solid line refers to the baseline equilibrium with labor tax rate $\tau_l = 0.15$ and a dashed line refers to an equilibrium with tax rate $\tau_l = 0.45$. High school workers from ability group 3 are the most productive ones, so the top solid and dashed lines describe ability group 3, while ability groups 4, 2 and 1 lie below them in descending order.

In the baseline equilibrium, the lifetime utility of high school workers is strictly concave in retirement age and all optima occur at a kink at the official retirement age 65, as marked

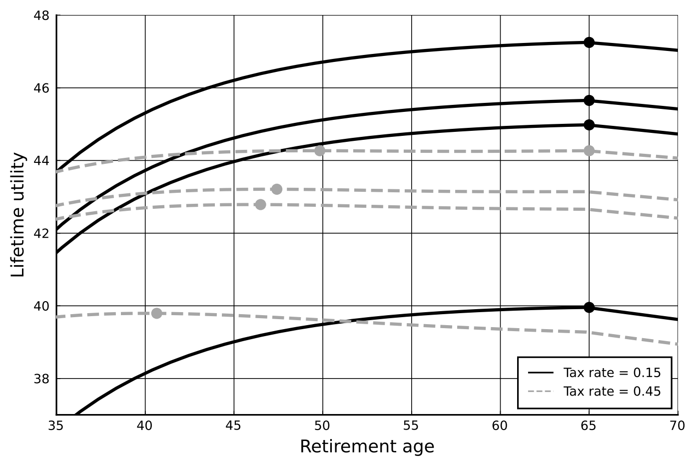

Figure 9: Lifetime utilities of high school workers by ability groups, conditional on working until the age on the horizontal axis, while optimally choosing consumption and human capital accumulation. For each ability group, the solid (dashed) line is computed using prices and lump-sum transfer from the tax-and-transfer equilibrium with labor tax rate $\tau_l = 0.15$ ($\tau_l = 0.45$). From top to bottom, ability groups appear in the order 3, 4, 2, and 1. A bullet shows an optimum, which is unique on each line except in the case of ability group 3 and $\tau_l = 0.45$ when there are two optima (on the top dashed line).

by a bullet on each solid line in Figure 9. In the equilibrium with $\tau_l = 0.45$, a non-concavity in the lifetime utility for ability group 3 results in two optima: one at the official retirement age 65 and another at an early retirement at age 50, respectively, as marked by two bullets on the top dashed line in Figure 9. The former (latter) optimum is associated with high (low) end-of-life human capital, as indicated by the dark solid (dashed) line for ability group 3 in Figure 7. While workers are indifferent between such two career strategies at tax rates in the range 0.42–0.52, market clearing pins down equilibrium *fractions* of high school workers in ability group 3 who adopt the two strategies, resulting in an average end-of-life human capital depicted by the light solid line for ability group 3 in Figure 7.

For college workers of different ability groups, Figures 10 and 11 depict end-of-life human capital stocks and retirement ages, respectively, as functions of the tax rate on labor income. Qualitatively, outcomes resemble those for high school workers in ability group 3. First, a college worker's end-of-life human capital initially increases with increases in the tax rate. For high school workers in ability group 3, their higher stocks of human capital offset lower stocks of college human capital of other workers. But unlike high school workers in ability group 3 who make up for other high school workers' shortening their career lengths and

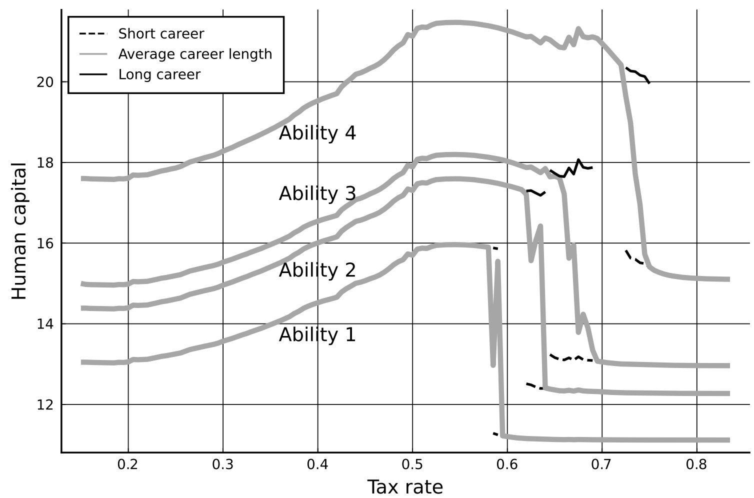

Figure 10: End-of-life human capitals of college workers by ability groups as functions the tax rate on labor income. As in Figure 7, dark solid and dark dashed lines for an ability group indicate that workers are indifferent between accumulating relatively high and low stocks of human capital, respectively, whereas the intermediate light solid line indicates a worker's average human capital in an equilibrium.

investing less in human capital, college workers' responses initially only make up for a falling college enrollment rate. Thus, at tax rates below 0.62, practically all college graduates continue to work until the official retirement age 65, while the college enrollment rate falls monotonically as the tax rate increases, as shown by the dark solid lines in Figures 2(b) and 3(a), respectively.[26] Second, like high school workers in ability group 3, college graduates of all ability groups eventually encounter an interval of tax rates over which they are indifferent between two very different career strategies, as indicated by Figures 10 and 11.

The preceding nonconvexities in the space of career strategies arise in the baseline economy when workers might choose a corner solution to retire at the official retirement age

[26] College graduates from the lowest ability group 1 do reduce their retirement ages and human capital accumulations just before τ_l reaches 0.60, as shown in Figures 10 and 11. However, at those tax rates, less than 0.1 percent of agents in ability group 1 attend college, so their effect on aggregate outcomes is small. This explains how there can be large swings in fractions of these workers who choose long and short career lengths, respectively, along a transition from a situation in which all of them choose long career lengths up and until tax rate $\tau_l = 0.58$ when all of them instead choose short career lengths at tax rates above $\tau_l = 0.595$ in Figure 11, with these changes not causing noticeable repercussions on equilibrium choices of career lengths and human capitals by workers of other types. Presumably, these swings are spurious outcomes as the computer algorithm seeks to divide an almost nonexistent category of workers, namely, college graduates in ability group 1, across two very different career choices between which they are indifferent over a small tax interval.

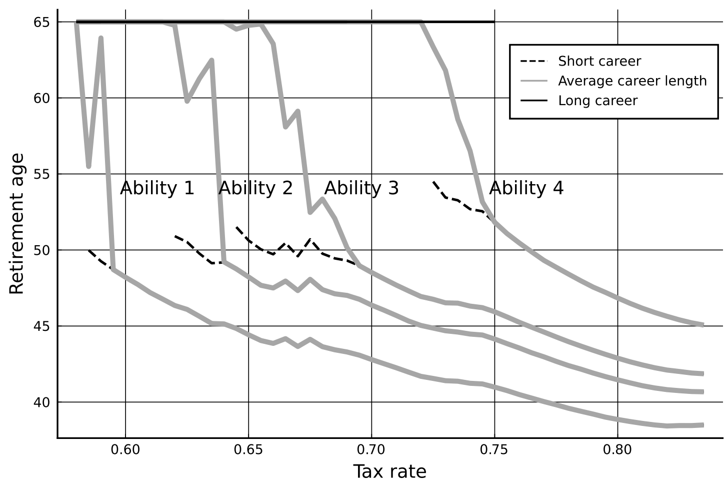

Figure 11: Retirement age of college workers by ability group as a function of the tax rate on labor income. Farthest to the left, all workers retire at the official retirement age 65. Next, for each ability group, there is a range of taxes over which workers are indifferent between retiring at age 65 and at an earlier age on the dashed line, whereas the light solid line shows the average retirement age in an equilibrium. Thus, when a light solid line merges with the dashed line, all workers of that ability group retire early.

because of an implicit extra tax wedge on labor income after that age. Nonconvexities can also arise under the social security reform when all workers are at interior solutions with respect to career length and also in a laissez-faire economy without any government intervention. The Ben-Porath human capital technology is the source of the nonconvexities. Gains from investments in human capital can be harvested only by choosing long careers. If government policies or the human capital technology make accumulating human capital less rewarding, an individual agent can confront a situation in which he or she chooses between a long labor market career with substantial investments in human capital or a much shorter career length with little or no investment in human capital. At high enough tax rates, all workers in the baseline economy choose not to accumulate human capital, making end-of-life human capitals in Figures 7 and 10 equal initial endowments of human capitals for all ability groups and schooling choices.

Despite discontinuities in an individual agent's labor market choices as functions of the tax rate on labor income, the equilibrium ratio of aggregate college to high school human capitals in Figure 3(b) is relatively smooth, and so are average retirement ages of college and high school workers in Figure 2(b). In a model with more heterogeneities across agents, such smoothness could emerge as higher tax rates induce more and more agents abruptly to alter their career choices. In our model with its limited heterogeneity in the form of only 8 pairs of of ability levels and schooling choices, the convexification of aggregate outcomes arises instead because of subsets of agents become indifferent between two very different labor market strategies, so that as the tax rate increases market clearing gradually adjusts equilibrium fractions of workers who adopt different strategies. The college enrollment rate in the economy with lump-sum rebates of tax revenues is an aggregate outcome that does not show smooth dependence on the tax rate. Thus, the solid line in Figure 3(a) levels out over the tax range 0.62–0.75 as college workers gradually switch from retiring at the official retirement age 65 to retiring earlier in Figure 11. Here an underlying monotone decline in the aggregate quantity of college human capital continues smoothly even though over the tax range 0.62–0.75, this decline no longer comes from a falling college enrollment rate, but instead from college workers choosing shorter career lengths and less human capital accumulation. When a tax rate equal to or exceeding 0.75 has induced all workers to retire early, the college enrollment rate resumes its decline and does so precipitously.

6 Taxation after social security reform

When conducting our Section 5 tax experiment in an economy under the Section 4 social security reform, we find that the Figure 12(a) Laffer curves closely resemble those for the economy under the baseline social security system in Figure 2(a). Nevertheless, there are differences in retirement age outcomes. The second column of Table 5 indicates that the social security reform induces college workers to retire *later*, on average at age 70.8, while high school workers choose to retire *earlier*, on average at age 63.9. These two outcomes appear at the farthest left points of dark and light lines, respectively, in Figure 12(b), i.e., at the baseline tax rate $\tau_l = 0.15$.

Under the social security reform, agents are not necessarily stuck at a corner solution that makes them retire at the official retirement age 65. There are two telltale differences between the economy under the baseline social security system in Figure 2(b) and the economy under the social security reform in Figure 12(b) when tax revenues from labor taxation above the baseline rate $\tau_l = 0.15$ are handed back as lump-sum transfers (solid lines). In the former economy, for small increases in taxation above the baseline rate $\tau_l = 0.15$, high school workers initially remain stuck at the official retirement age. But in the latter economy, being at interior solutions with respect to career lengths, high school workers' average retirement age decreases with the first increments in the tax rate. In the former economy, until fairly

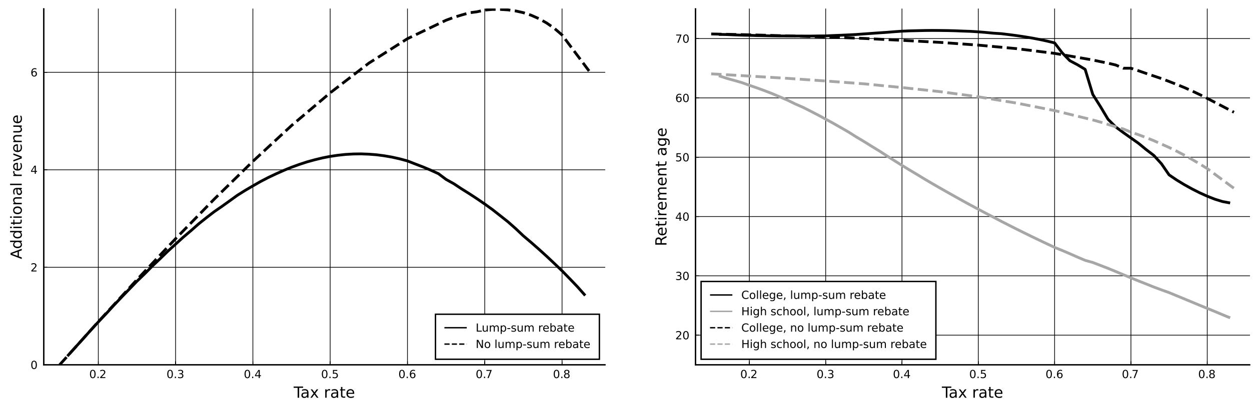

Figure 12: Laffer curves and retirement ages under the social security reform as functions of the tax rate on labor income and whether or not tax revenues from labor taxation above the baseline rate $\tau_l = 0.15$ are distributed as lump-sum transfers. In Panel A, solid and dashed lines indicate Laffer curves with and without lump-sum rebates, respectively. In Panel B, dark and light solid (dashed) lines are average retirement ages of college and high school workers, respectively, with (without) lump-sum rebates.

high tax rates college workers increase their lifetime accumulations of human capital. By way of contrast, in the latter economy, because they are at interior solutions with respect to their choices of career lengths, college workers also increase their average retirement ages over a mid-range of tax rates in Figure 12(b).

Eventually, the tax rate becomes so high that all workers, both in the economy under the baseline social security system and in the economy under the social security reform, choose to retire before the official retirement age. For tax rates so high, equilibrium outcomes in the two economies are identical. That occurs when the tax rate rises above $\tau_l = 0.74$ ($\tau_l = 0.72$) with (without) lump-sum rebates of tax revenues, so that the associated lines in Figures 2 and 12 are identical beyond those tax rates. Over this range of tax rates in the economy under the baseline social security system, the implicit extra tax wedge on labor income after age 65 becomes irrelevant and workers' choices of career lengths are the same as they would be in an economy under the social security reform without the extra tax wedge.

Despite these differences, if we were to conceal the numbers on the vertical axis in Figures 2(b) and 12(b), profiles of college workers' average retirement age as functions of the tax rate would appear to be very similar. An official retirement age at 65 "anchors" career lengths of college workers in Figure 2(b), while something else apparently anchors career lengths in Figure 12(b). That "something else" is a combination of the earnings dynamics of college workers relative to high school workers, which in our model causes college workers to choose longer careers, and our specification of the fixed disutility B of working and the human capital depreciation schedule in Figure 1 (solid line). These model features induce college workers to retire at an age at which human capital depreciates rapidly. Consequently, under the social security reform a "wedge" coming from old-age human capital depreciation requires relatively strong forces to dislodge college workers' choices of career lengths, producing effects similar to those induced by the implicit extra tax wedge on labor income after the official retirement age under the baseline social security system.

7 Aggregate labor supply elasticity

In view of agents' dichotomous choices of whether to work full-time or not at all in a period, we let an employment-to-population ratio Ω represent aggregate labor supply, as depicted in Figure 13(a) for the Section 5 tax experiments. To measure sensitivity of Ω to labor taxation, we compute the aggregate labor supply elasticity Ξ with respect to the net-of-tax rate $1-\tau_l$:

$$\Xi = \frac{\partial \Omega}{\partial (1-\tau_l)} \frac{1-\tau_l}{\Omega}, \tag{22}$$

as reported in Figure 13(b).[27] The solid lines in the two panels of Figure 13 are for the economy in which tax revenues from labor taxation above the baseline rate $\tau_l = 0.15$ are handed back as equal lump-sum transfers to agents, while the dashed lines describe the economy without such lump-sum rebates.

Consistent with outcomes described in Section 5, the aggregate labor supply elasticity is suppressed in the economy without lump-sum rebates. After a slow, gradual increase, there is a big increase in the elasticity when the equilibrium interest rate in Figure 4(a) (dashed line) begins to increase exponentially near tax rate $\tau_l = 0.7$. This is because of physical capital gets scarce. High tax rates distort labor supplies more in the economy with lump-sum rebates. After a brief initial range of a zero aggregate labor supply elasticity because all workers are stuck at a corner solution of retiring at the official retirement age 65, the elasticity quickly increases above 1, where it levels off around 1.2 until increasing again after tax rate $\tau_l = 0.7$.[28]

[27] Because of small, unnoticable fluctuations in the employment-to-population ratio Ω in Figure 13(a), we use a 7-point centered moving average of Ω to smooth the computations of the elasticity Ξ in Figure 13(b). Instead of truncating the elasticity calculations at the end points of the tax range, we use fewer points to form the moving averages of Ω around the end points.

[28] When preferences are the same as our utility function (2) in the framework of Ljungqvist and Sargent (2014, p. 9, eq. (26)), the corresponding aggregate labor supply elasticity is analytically equal to one, regard-

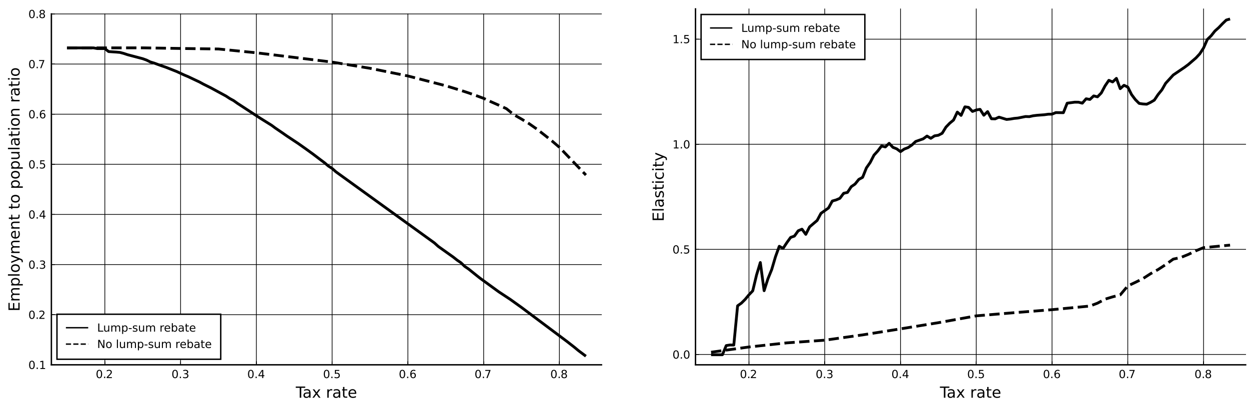

Figure 13: Employment to population ratio (Panel A) and aggregate labor supply elasticity (Panel B) as functions of the tax rate on labor income and whether or not tax revenues from labor taxation above the baseline rate $\tau_l = 0.15$ are handed back as equal lump-sum transfers to agents. In both panels, solid and dashed lines display outcomes with and without lump-sum rebates, respectively.

Since college workers are stuck at the corner solution of retiring at the official retirement age 65 practically until the tax rate has risen to $\tau_l = 0.6$, the approximately flat aggregate labor supply elasticity of around 1.2 in the economy with lump-sum rebates might seem surprising. But decreases in college enrollment rates at lower tax rates in Figure 3(a) (solid line) mean that agents who would have become college workers at lower tax rates have now chosen to be high school workers and to retire at the earlier ages chosen by high school workers.

If we compare outcomes in Figure 13(a) with corresponding employment-to-population ratios in the Section 6 economy under social security reform, the Section 6 economy differs only in having a labor supply that is initially slightly elevated. This make aggregate labor supply elasticities be similar in the two economies.

8 Inequality

Government policies that affect choices of schooling, on-the-job accumulation of human capital, and retirement ages have distributional consequences. Our model teaches us that the

less of the exponent on past work experience in a power function that determines the current wage in the learning-by-doing technology. We obtain a similar elasticity of around 1.2 over a substantial middle range of tax rates in a time-averaging version of HLT's growth model with a Ben-Porath human capital technology and several other features that differ from the Ljungqvist-Sargent framework.

ultimate effects of tax rate changes on inequality are intermediated through changes in equilibrium rental rates of the human capital stocks that are accumulated by different types of agents, changes that are themselves caused by equilibrium responses by all eight types of agents in choosing whether to go to college, along with their career lengths and human capital accumulation decisions. These complicated dynamic general equilibrium interactions set the stage for possibly unintended consequences of policy changes that might at first glance aim to redistribute resources from more advantaged to less advantaged workers, for example, an increase in the tax rate on labor income that raises revenues that are used to raise lump sum transfers to agents of all types.

In this spirit, we want to study effects on different ability groups and across schooling levels of the Section 5 tax rate changes that are accompanied by lump-sum transfers of tax revenues. To proceed, we compute two measures of inequality, one cast in terms of present values of lifetime labor earnings, the other in terms of lifetime utility.

8.1 Labor income inequality

We want to quantify how inequalities in labor incomes and welfares differ across equilibria indexed by different tax rates. Sources of labor income differences include four exogenous ability groups and two endogenous choices of schooling. An additional source of income differences arises when, as described in Section 5.2, there exist equilibria in which some agents with the same abilities and schooling levels are indifferent between two distinct career strategies. For those equilibria, we compute the average labor income for agents of such an ability group and schooling level, using as weights the equilibrium fractions of agents who pursue different career strategies. To measure present values of lifetime labor earnings at a given tax rate, we discount them at the equilibrium interest rate. We report those present values relative to an economy-wide average present value of lifetime labor earnings at that tax rate.

Figure 14(a) shows our measures of income inequality across ability groups for high school workers at different tax rates. Outcomes below tax rate $\tau_l = 0.5$ closely resemble end-of-life human capitals of high school workers portrayed in Figure 7. Likewise, the initial increases in relative labor earnings of college workers in Figure 14(b) resemble increases in their end-of-life human capitals portrayed in Figure 10, including abrupt declines for each ability group of college workers as increases in the tax rate induce them to switch from strategies of long careers and substantial human capital accumulations to shorter careers and little or

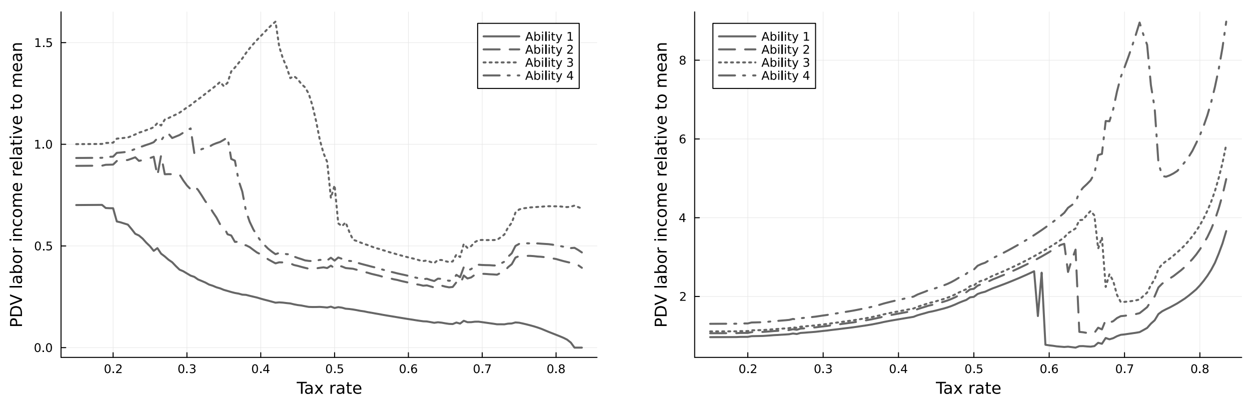

Figure 14: Labor income inequality across ability groups of high school workers (Panel A) and college workers (Panel B) as functions of the tax rate on labor income. We measure income as a present value of lifetime labor earnings discounted at the equilibrium interest rate for each tax rate, which we then express relative to the average of the present values of lifetime labor earnings in an economy for that tax rate. When agents with the same abilities and schooling levels are indifferent between two distinct career strategies, we report an average for that ability group and schooling level.

no human capital accumulations. Effects on equilibrium wages of those switches of career strategies by all ability groups of college workers account for the gains made by most high school workers in Figure 14(a) in terms of their relative labor earnings at the top range of tax rates. A notable exception is the lowest ability group of high school workers. The decline in their relative labor earnings is explained by large decreases their career lengths in Figure 8 that have them not working at all at the highest tax rate, making their present values of lifetime labor earnings fall to zero in Figure 14(a). Those workers decide to rely entirely on government support in the form of a lump-sum transfer of tax revenues and a social security benefit collected after age 65.

After the striking declines in relative labor earnings of each ability group of college workers just described, there is another surge in their labor earnings at the top range of tax rates in Figure 14(b). These sharply increasing relative labor earnings are driven by increases in the price of college human capital relative to the price of high school human capital, as indicated by the solid line in Figure 5. Equilibrium wages adjust to compensate college workers for the nonpecuniary costs that they incur to attend college. As the tax rate increases, two countervailing forces determine how much the labor income of a college worker must rise relative to that of a high school worker. On the one hand, as equilibrium college enrollment rates decline in Figure 6, the nonpecuniary college cost falls for a marginal agent who attends college from each ability group. That tends to lower required compensating earnings differential. On the other hand, as the tax rate increases, a worker's take-home labor earnings become smaller, which tends to increase how much higher the gross income of a college worker must be relative to what he would have earned as a high-school-educated worker. Evidently, at the top range of tax rates, the latter force is much stronger, so a college worker must earn several times more than a high school worker.

Figure 15 shows how Gini coefficients vary across equilibria at different tax rates.[29] The Gini coefficient remains almost constant for the first five percentage points increase over the baseline tax rate $\tau_l = 0.15$. This is a consequence of all workers continuing to work until the official retirement age 65, together with those tax rate changes inducing only small changes in decisions about enrolling in college and accumulating human capital. As the tax rate increases more, there is a nearly linear one-to-one relationship between increases in the tax rate and the Gini coefficient until the tax rate reaches $\tau_l = 0.65$, when the relationship between the Gini coefficient and the tax rate flattens as ability groups 2 and 3 of college workers switch from pursuing ambitious career strategies to choosing shorter careers with little or no accumulating of human capital.[30] The flattening ends with a significant drop of the Gini coefficient when ability group 4 of college workers, the highest earners, also switch to pursuing shorter careers and accumulating less human capital. The final surge in all college workers' earnings in Figure 14(b) makes the Gini coefficient increase again. The increase is steeper than before the plateau, but not as dramatic as we might have anticipated based on the sharply increasing college earnings in Figure 14(b). This happens because college workers are becoming a smaller fraction of the work force, as suggested by declining college enrollment rates and the sharply declining ratio of college to high school human capitals in the production of goods, as indicated by the solid lines in Figures 3(a) and 3(b), respectively.

8.2 Welfare inequality

Welfare comparisons across schooling levels are complicated by the fact that schooling level is an endogenous state, chosen by agents. To recognize that endogeneity we use an *ex post-ex ante* welfare measure proposed by Holter, Ljungqvist, Sargent, and Stepanchuk (2025) to evaluate how our Section 5 tax experiments affect welfare inequality across ability groups and

[29]The Gini coefficient in Figure 15 also accounts for any instances of income inequality among workers of the same ability and schooling who are indifferent between and pursue different career strategies.

[30]As described in footnote 26, the same switch of college workers of ability group 1 is inconsequential for aggregate outcomes since they are so few in number.

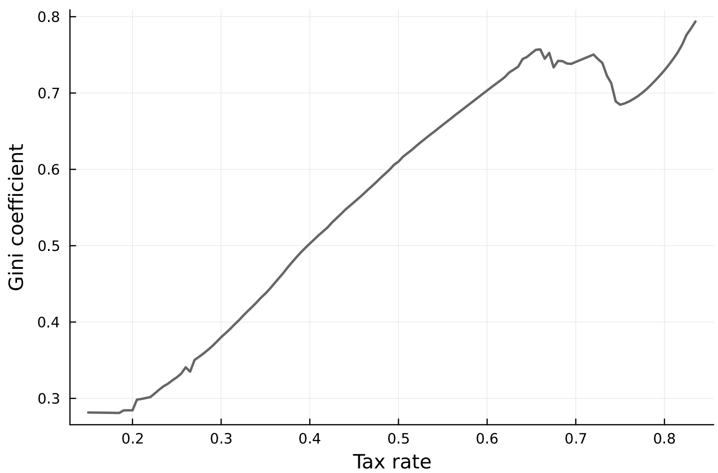

Figure 15: Gini coefficient as a function of the tax rate on labor income.

schooling levels. For each ability group, conditional on *ex post* choices of schooling levels, the *ex ante* object is the average lifetime utility of all agents with that ability and schooling. We measure welfare in terms of consumption equivalents relative to the unconditional lifetime utility of an agent who has just entered economic life as an 18-year old[31] under a veil of complete ignorance about the agent's ability and nonpecuniary cost of attending college. Figure 16 reports those *ex post-ex ante* welfare measures.[32]

For each ability group, Figure 16(a) shows welfare conditional on not attending college in different equilibria indexed by the labor tax rate. Note that this *ex post-ex ante* welfare measure entails no averaging, since all high school workers of the same ability attain the same welfare level. Furthermore, the only differences in primitives among high school workers' ability groups are two human capital parameters: initial endowments of high-school human capital $H^1(\cdot)$, and the productivity factor $A^1(\cdot)$ in the technology for accumulating additional human capital on the job. Since the ability-specific pairs of parameters in Table 1 can be ranked unequivocally from the highest to the lowest, it follows that the ability-specific welfare levels of high school workers inherit that same ordering, so ability group 3 attains the highest welfare followed by groups 4, 2, and 1, as depicted in Figure 16(a). The idiosyncratic

[31] This is the age at which a new cohort enters the economy.

[32] Incidentally, the Section 8.1 situation in which some agents with the same abilities and schooling levels are indifferent between pursuing different career strategies does not complicate things here: their indifference means that such workers would obtain the same lifetime utility by choosing either of those career strategies. Note that this is not true for the labor earnings that we used to measure income inequality studied in Section 8.1.

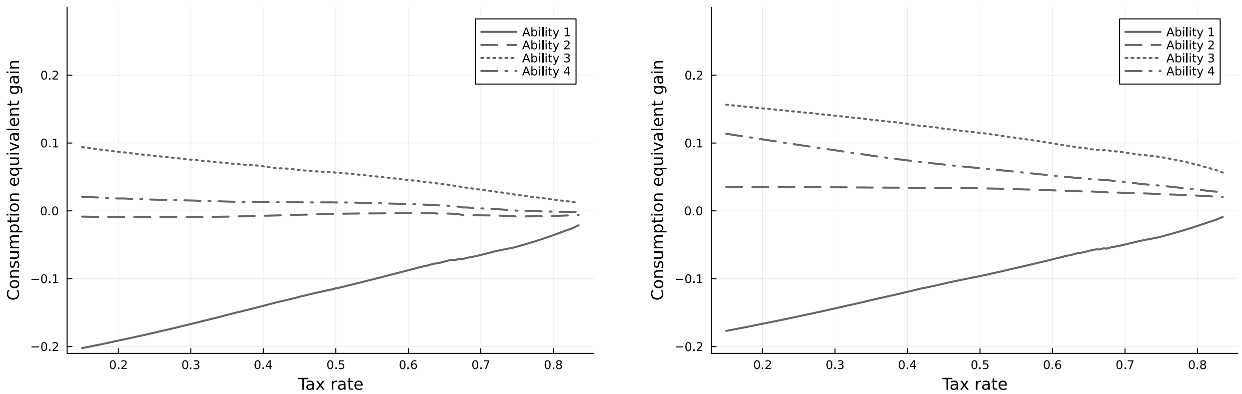

Figure 16: *Ex post-ex ante* welfare measures by ability group and schooling level as functions of the tax rate on labor income. Panels A and B show high school and college workers, respectively. Consumption equivalents are relative to the unconditional lifetime welfare of a 18-year old entering the economy under a veil of complete ignorance.

nonpecuniary cost of attending college complicates calculating the welfare of college workers in Figure 16(b), so it is instructive to perform this calculation in two steps. In our first step, we ignore the nonpecuniary cost of attending college in the following way. Since that additively separable disutility is a sunk cost for those who choose to become college workers, the *remainder* optimization problem is similar to the problem of high school workers; hence college workers of the same ability attain a common *remainder* lifetime welfare, excluding the nonpecuniary cost of attending college. Furthermore, aside from the nonpecuniary costs of attending college, it happens that the only primitives that differ among ability groups of college workers, is a pair of human capital parameters $(H^2(\cdot), A^2(\cdot))$. These ability-specific pairs in Table 1 can once again be ranked unequivocally from the highest to the lowest. But now the ordering is that ability group 4 attains the highest *remainder* welfare, followed by groups 3, 2, and 1. Next, in our second step, we integrate over the nonpecuniary cost of attending college among workers in each ability group who have chosen to become college workers. This cost in Table 1 is the second highest for ability group 4 and the lowest for ability group 3. Evidently, these differences in costs serve to reverse the top ordering from *remainder* welfares to become that of total welfares in Figure 16(b), where ability group 3 now comes out on top.

Trivially, for each ability group, the *ex post-ex ante* welfare of college workers must be higher than that of high school workers. A marginal college worker in each ability group is indifferent between becoming a college worker or being a high school worker and attaining a common ability-specific lifetime high-school-worker welfare. Hence, for each ability group,

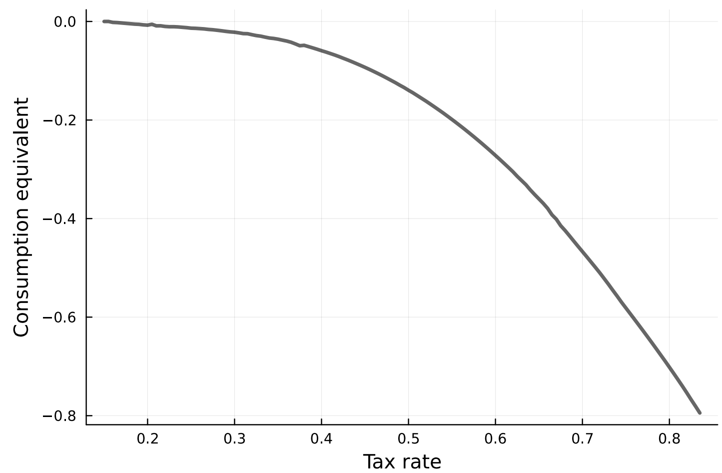

Figure 17: Efficiency loss of taxation measured in terms of the unconditional lifetime welfare of a 18-year old entering the economy under a veil of complete ignorance, as a function of the tax rate on labor income. Consumption equivalents are relative to the unconditional lifetime welfare of an 18-year old entering the economy with the baseline tax rate $\tau_l = 0.15$ under a veil of complete ignorance.

the lower nonpecuniary costs of attending college among the inframarginal college workers makes the *ex post-ex ante* welfare of college workers be strictly higher than that for high school workers. Another notable pattern in Figure 16 is that higher rates of taxation and accompanying lump-sum transfers cause welfare inequality to shrink. Not surprisingly, such redistribution is associated with a loss of efficiency quantified in Figure 17. Here we adopt a standard *ex ante* approach that expresses the loss of a 18-year old entering an economy with elevated taxation under a veil of complete ignorance about his ability and nonpecuniary cost of attending college as a consumption equivalent relative to instead having entered an economy with the baseline tax rate $\tau_l = 0.15$. Figure 17 shows that this loss is initially modest, but that it increases quickly as the tax rate is raised above $\tau_l = 0.4$.

9 Concluding remarks

Our inclusion of endogenous career lengths and indivisible labor substantially enriches HLT's general equilibrium analysis of labor market outcomes with respect to schooling choices and on-the-job human capital accumulation. As a starting point, we show how the addition of a social security system can rationalize HLT's entire analysis when career lengths are endogenous. If the retirement system imposes an implicit extra tax wedge on labor income after an official retirement age in the form of lost social security benefits, agents who are heterogeneous in abilities and schooling levels might nevertheless choose corner solutions at an official retirement age.

HLT's labor force composition of college and high school graduates highlights earnings profiles as determinants of career length. In a learning-by-doing setup, Ljungqvist and Sargent (2014, sec. 3) show that the more elastic is an earnings profile to accumulated working time, the longer is a worker's career. We have shown how this outcome extends to HLT's Ben-Porath human capital technology where the technology of college workers is estimated to be more productive and hence, college workers are prone to choose longer careers than high school workers. Longer careers of college workers emerge from under a social security reform that removes the implicit extra tax wedge on earnings after the official retirement age. Because we assume preferences that are consistent with balanced growth, it is not the level of earnings that matter but only terms on which more human capital can be accumulated on a job.

In a companion paper, Heckman, Lochner, and Taber (1998b) compared partial- and general-equilibrium effects of tax reforms that favor investments in human capital. Given a fixed interest rate and skill prices, while college enrollment and on-the-job human capital accumulation can increase markedly in a partial equilibrium, responses are much diminished in a general equilibrium in which a higher interest rate emerges from a relative scarcity of physical capital. That interest rate channel is very much present in our tax experiments, while general-equilibrium dynamics coming from endogenous career lengths further augment it. When tax revenues are handed back as lump-sum transfers, changes in the labor tax rate bring stark interdependencies across ability and schooling levels. Until fairly high tax rates on labor income, the negative impact on college human capital comes from falling college enrollment, while those who do graduate from college continue to work long careers and to invest in human capital on the job. In contrast, high school workers shorten career lengths and reduce investments in human capital, especially those in the lowest ability groups. Thus, those high labor tax rates exacerbate a labor market bifurcation between highly active workers and some who have been sidelined or marginalized.

Bifurcation manifests itself as growing lifetime labor income inequality that reflects high school workers with poor labor market prospects shortening their career lengths as well as decisions by workers with more productive Ben-Porath technologies to increase their human capital investments in response to higher taxes. The most able high school workers respond in this way, as do college graduates. Unintended general equilibrium effects of the tax-and-transfer policy marginalize large groups of less advantaged workers and simultaneously motivate advantaged workers to increase their contributions to production. Despite that growing *income* inequality, because tax revenues are rebated lump-sum, measures of *welfare* conditioned on schooling levels become more and more *equal*. Still, a declining unconditional lifetime welfare of a 18-year old entering the economy under a veil of complete ignorance shows that convergence of lifetime welfares across agents is attained at a growing efficiency cost. That efficiency cost accelerates after distortions reach "tipping points" that we describe next.

We encountered a perhaps neglected property of a Ben-Porath human capital technology in the form of nonconvexities in choice sets for career strategies. For investments in human capital to pay off, an investor needs to choose a long enough career. That complicates responses to changes in the disutility of working, productivity of the human capital technology, and tax rates. We discovered that the optimal response to an incremental change in determinants can be discontinuous and possibly even provoke large shifts in career length and human capital investments. For example, in our tax experiments with lump-sum rebates of tax revenues, such big responses are made by the most able high school workers and college workers of all ability groups who will, at some incremental tax increase experience a "tipping point" that takes the form of a discrete change from a long career with substantial human capital investments to a significantly shorter career with much less on-the-job human capital accumulation.

Thus, our time-averaging version of the HLT model brings out challenges for economic policy making. Some of our social security reforms and tax-and-transfer arrangements increase the hazard of a "dual labor market" marked by different labor market participation rates across schooling and ability groups. Agents with steeper earnings profiles, like college workers, are prone to participate more robustly in the labor market. The steepness of an agent's earnings profile depends on both his Ben-Porath technology and his choice of how much human capital to accumulate on the job. This affects the efficacy of tax-and-transfer schemes. If government policies eventually bring high enough tax wedges and other distortions, then even those agents having the most productive human capital technologies who have historically supplied more labor more robustly become a source of labor market fragility and might dramatically shift their career strategies toward earlier retirements and less human capital accumulation. A prospective implosion of economic activity raises the stakes for avoiding policies that erode labor market participation among substantial proportions of the labor force.

10 Post scriptum

Fan, Seshadri, and Taber (2024) endogenize career lengths in a Ben-Porath human capital model that they estimate using data on male high school graduates from the Survey of Income and Program Participation (SIPP) and other sources. They target moment conditions across ages 22-65 for employment rates, wages, and consumption; social security benefit application rates for ages 62-70; overall transition probabilities between working and not working averaged between ages 35 and 50; and wage changes after nonemployment spells averaged between ages 41 and 65. To supplement Fan et al.'s aggregative perspective, we first study the diversity of workers' lifetime experiences in their model, then use them to compare forces at work in Fan et al.'s model and ours.

In terms of our notation, nine types of workers indexed by $\theta \in \{1, 2, \dots, 9\}$ live in the Fan et al. model. Types differ in their disutilities of work $B(\theta)$, abilities to learn $A(\theta)$, and initial human capitals $H(\theta)$.[33] Because the latter two characteristics are highly positively correlated, pairs of productivity characteristics $(A(\theta), H(\theta))$ can be ranked unambiguously from lowest to highest. Disutility of work takes on three values, $B^L < B^M < B^H$, and for each value of B^i, there are three productivity levels that we present in ascending order, $(A^i_L, H^i_L) < (A^i_M, H^i_M) < (A^i_H, H^i_H)$. Furthermore, Fan et al.'s estimates of $(A^L_i, H^L_i) > (A^M_i, H^M_i) > (A^H_i, H^H_i)$ for $i = L, M, H$ indicate that disutility of work and productivity are highly negatively correlated. Thus, we can represent the *ex ante* heterogeneity of people in Fan et al.'s model with a 3×3 matrix with row entries being disutilities of work and column entries indicating L(ow), M(edium), and H(igh) productivities. See Table 6, where position (i, j) refers to worker type $(B^i, (A^i_j, H^i_j))$.

In response to our request, Fan et al. computed an average number of periods (years) working (a.k.a. career length) for each type of worker that we report in Table 6. Evidently, along each column average career length decreases as the disutility of work increases. As for productivities across worker types, we recognize dissimilarities when we compare Fan et al.'s model and ours. Our finding that college workers choose longer careers than high school workers can be interpreted with assistance from the learning-by-doing setup of Ljungqvist

[33] Actually, while $B(\theta)$ and $A(\theta)$ assume type-specific values, Fan et al. specify that the initial human capital of a worker of type θ is a function of $B(\theta)$, $A(\theta)$, and an i.i.d. random variable. But since that random variable has a very small variance, we simplify our characterization of *ex ante* heterogeneity by lumping agents together under type-specific means of initial human capital, denoted by $H(\theta)$.

Table 6: Average career length $\hat{n}(i,j)$ by worker type (B^i, (A^i_j, H^i_j))

	Low productivity $\hat{n}(i,L)$	(A^i_L, H^i_L)	Medium productivity $\hat{n}(i,M)$	(A^i_M, H^i_M)	High productivity $\hat{n}(i,H)$	(A^i_H, H^i_H)
Disutility of work						
$B^L = 0.0011$	57.6	(1.63, 9.5)	47.9	(2.67, 17.2)	40.5	(3.70, 31.2)
$B^M = 0.0036$	50.8	(0.85, 6.2)	40.5	(1.88, 11.2)	33.3	(2.92, 20.4)
$B^H = 0.0120$	30.9	(0.06, 4.0)	32.9	(1.10, 7.4)	26.9	(2.13, 13.4)

Notes: Our parameters (B, A, H) map into Fan et al.'s parameters (a_0, π, H_0) as follows. Our disutility of work $B = \exp(a_0)$, where $a_0 \in \{-6.84, -5.63, -4.42\}$. Our ability to learn $A = \pi$, with values as shown in this table. Our initial human capital H of a worker type in this table is the average of Fan et al.'s H_0 across all workers with the same B and A, as explained in footnote 33.

and Sargent (2014). In their model, the more elastic is an earnings profile to accumulated working time, the longer is a worker's career.[34] This force is largely absent from the model of Fan et al. with its imposition of identical exponents in Ben-Porath technologies across all workers. Furthermore, notice how differences in a multiplicative factor A don't affect career lengths in the Ljungqvist and Sargent model. These discrepancies across models brings out the importance of an assumption made both in our paper here and by Ljungqvist and Sargent that preferences are consistent with balanced growth: income and substitution effects of higher wages cancel, so higher wages leave labor supplies unchanged. Since Fan et al. do not impose that restriction on preferences, we suspect that the sharp reductions in career lengths in response to higher productivity levels in the first two rows of Table 6 are due to income effects.

We also suspect that time averaging strengthens income effects. Besides the dramatic declines in career lengths in Table 6, another analytical result that prevails when preferences are consistent with balanced growth is pertinent. In particular, Ljungqvist and Sargent (2014, sec. 2.1) analyzed how initial assets affect optimal career lengths. For example, they showed that with a flat earnings-experience profile an optimal career length at an interior solution equals the same optimal career length that would have been chosen with zero initial assets minus a deduction that increases linearly in initial assets, so that labor supply becomes zero when initial assets exceed the lifetime earnings that would have been gotten under the

[34] Ljungqvist and Sargent (2014) analyze a nonstochastic economy in continuous time with a utility functional whose felicity function is given by (1) and (2). They assume that the market interest rate and the subjective discount both equal zero.

career length without any initial assets times the intertemporal elasticity of substitution in consumption ($1/\gamma$). Such examples of large effects on career lengths indicate that income effects are strong in models of time averaging, an idea that we intend to investigate further.

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A Computational details

A.1 Solving an agent's problem

Conditional on a retirement age, we solve an agent's problem using a "shooting algorithm", exactly as in the original HLT paper.

For a household of age n, type θ, schooling level S, savings K, human capital H, who will retire at age $\hat{n}$, the recursive problem is:

$$V_n^{\hat{n}}(H,K,S,\theta) = \max_{C,I,K',H'} \Big[U(C,\omega_n)) + \delta V_{n+1}^{\hat{n}}(H',K',S,\theta)\Big] \tag{23}$$

$$\begin{aligned} &\text{subject to} \\ C + K' &= K(1+(1-\tau_k)r) + \omega_n(1-\tau_l-\tau_p)R^S He(n)(1-I) \\ &\quad + (1-\omega_n)(1-\tau_l)\mathbb{1}(n \geq \eta_p)P, \\ H' &= H + \omega_n A^S(\theta) I^{\alpha_S} H^{\beta_S}, \end{aligned}$$

where $\omega_n = 1$ for $n < \hat{n}$, and 0 otherwise. The FOCs for this problem are:

$$[C]: \quad U_c(C_n,\omega_n) = \lambda_n \tag{24}$$

$$[I]: \quad \lambda_n\omega_n(1-\tau_l-\tau_p)R^S H_n e(n) = \mu_n\omega_n A^S(\theta)\alpha_S I_n^{\alpha_S-1}H_n^{\beta_S} \tag{25}$$

$$[H']: \quad \delta V_{H,n+1}^{\hat{n}} = \mu_n \tag{26}$$

$$[K']: \quad \delta V_{K,n+1}^{\hat{n}} = \lambda_n \tag{27}$$

where λ_n and μ_n are the Lagrange multipliers associated with the budget constraint and the human capital accumulation equation. The envelope conditions are:

$$V_{H,n}^{\hat{n}} = \lambda_n\omega_n(1-\tau_l-\tau_p)R^S e(n)(1-I_n) + \mu_n[1+\omega_n A^S(\theta)I^{\alpha_S}\beta_S H_n^{\beta_S-1}] \tag{28}$$

$$V_{K,n}^{\hat{n}} = \lambda_n(1+(1-\tau_k)r) \tag{29}$$

We can use these conditions to find an Euler equation for consumption:

$$U_c(C_n,\omega_n) = \delta(1+(1-\tau_k)r)U_c(C_{n+1},\omega_{n+1}) \tag{30}$$

We can also get the following equation defining the evolution of μ_n:

$$\mu_n = \delta[\lambda_{n+1}\omega_{n+1}(1-\tau_l-\tau_p)R^S e(n+1)(1-I_{n+1}) + \mu_{n+1}[1+\omega_{n+1}A^S(\theta)I_{n+1}^{\alpha_S}\beta_S H_{n+1}^{\beta_S-1}]] \quad (31)$$

Given these conditions, for any guess of terminal values of human capital and savings, the above equations allow us to solve the household's problem backwards to the initial period. We can then iterate until the initial values of human capital and savings are equal to their true values. A detailed description of the algorithm is as follows:

1. Guess levels of human capital and savings for period $\overline{\eta}$.

2. In the final period the household will consume all available resources, therefore:

$$C_{\overline{\eta}} = K_{\overline{\eta}}(1+(1-\tau_k)r) + P$$

   Solve the consumption Euler equation backwards to find $\{C_n\}_{n=\underline{\eta}}^{\overline{\eta}}$. Use the consumption FOC to find $\{\lambda_n\}_{n=\underline{\eta}}^{\overline{\eta}}$.

3. In the final period of working life, optimality implies that $I_{\hat{n}} = 0$ and $\mu_{\hat{n}} = 0$, which implies that $H_{\hat{n}} = H_{\overline{\eta}}$.

4. Set $i = 1$.

5. Given the values of variables for period $\hat{n} - i + 1$, use the equation for the evolution of μ_n to solve for $\mu_{\hat{n}-i}$.

6. Use the following two equations to solve for $H_{\hat{n}-i}$ and $I_{\hat{n}-i}$:

$$H_{\hat{n}-i+1} = A^S(\theta)I_{\hat{n}-i}^{\alpha_S}H_{\hat{n}-i}^{\beta_S} + H_{\hat{n}-i} \quad (32)$$
$$\lambda_{\hat{n}-i}(1-\tau-\tau_p)R^S H_{\hat{n}-i}e(\hat{n}-i) = \mu_{\hat{n}-i}A^S(\theta)\alpha_S I_{\hat{n}-i}^{\alpha_S-1}H_{\hat{n}-i}^{\beta_S}$$

7. If $i < \hat{n} - \underline{\eta}$, set $i = i + 1$ and return to step 5. Otherwise, move to step 8.

8. Given the series for $\{C_n\}_{n=\underline{\eta}}^{\overline{\eta}}$, $\{H_n\}_{n=\underline{\eta}}^{\overline{\eta}}$, and $\{I_n\}_{n=\underline{\eta}}^{\overline{\eta}}$, use the budget constraint to solve for $\{K_n\}_{n=\underline{\eta}}^{\overline{\eta}}$.

9. If $K_{\underline{\eta}}$ and $H_{\underline{\eta}}$ differ from required values, update guesses of $K_{\overline{\eta}}$ and $H_{\overline{\eta}}$ and return to step 2.

To solve for the optimal retirement age, we simply deploy the shooting algorithm for every possible retirement age and then select the retirement age that maximizes lifetime utility.

Regarding agents who choose to attend college, the expressions above for the four years spent in college should be modified as described in Section 2.2.

A.2 Solving for a general equilibrium

In this paper, we run a number of experiments that require us to find new equilibrium prices. The three prices are (r, R^1, R^2) which represent the real interest rate and the skill prices of high school and college human capital, respectively. They are pinned down by the derivatives of the production function in (15) - (17). In some experiments, we also increase the labor income tax rate above our baseline of 0.15 and allow the additional revenue to be rebated back to households in the form of a lump-sum transfer P_{LS}. In those experiments, we also need to find the lump-sum transfer that satisfies the government budget constraint in addition to the three prices.

This general equilibrium problem necessitates an outer loop over the shooting algorithm described in the previous section. Because of the discrete nature of some of the decisions in our model (such as retirement age and the decision to work or not), commonly-used nonlinear solvers run into issues converging on a price vector. One way in which we deal with these discontinuities is by using a more robust price-finding algorithm. Our strategy relies on a combination of the Nelder-Mead algorithm from the Julia package `Optim.jl` and a dampened bisection algorithm. This combination is useful because the bisection algorithm works very well if the equilibrium mapping is continuous. The incorporation of Nelder-Mead enables us to be more "robust" to discontinuities, even though it is less efficient.

Given an initial guess for the price vector and any lump-sum transfer, our algorithm proceeds as follows:

1. Run Nelder-Mead for 100 iterations.

2. Use the outcome from Nelder-Mead as the initial guess to the dampened-bisection algorithm, which runs for 200 iterations. We update the price vector guess according to the following:

$$P_{LS}^{new} = 0.9 P_{LS}^{old} + 0.1 P_{LS}^{out}$$
$$r^{new} = 0.9 r^{old} + 0.1 r^{out}$$

$$
\begin{aligned}
(R^1)^{new} =& 0.98(R^1)^{old} + 0.02(R^1)^{out} \\
(R^2)^{new} =& 0.98(R^2)^{old} + 0.02(R^2)^{out}
\end{aligned}
$$

where the variables with superscript *out* represent the prices or transfer that would satisfy the first-order conditions or government budget constraint given the aggregates that result from the shooting algorithm. The extent of dampening on the skill prices (R^1, R^2) is different from that of (r, P_{LS}) because the aggregates in the model respond less continuously to changes in the skill prices.

3. When the dampening algorithm reaches 100 iterations, it is restarted from the best guess price vector so far, and the dampening parameters are halved.

4. If the outcome is still not satisfactory, steps 1 through 3 can be repeated using the outcome of the bisection algorithm as the initial guess for Nelder-Mead.

5. The algorithm stops if at any point the following residual is less than `1e-8`:

$$
\text{residual} = \sum_{i=1}^{4} \left(\frac{x_i^{old} - x_i^{out}}{x_i^{old}} \right)^2
$$

where the $\{x_i\}_{i=1}^4$ represent $\{P_{LS}, r, R^1, R^2\}$.

A.3 Modifications due to non-convexities

The algorithm in the previos section goes a long way towards solving for a general equilibrium in the face of the non-convexities in the model. However, it does not go far enough at certain points in our tax experiments. There are two additional modifications we make in order to overcome these challenges. One is primarily computational and the other has economic substance. We turn to the computational modification first.

A.3.1 Continuous choice of retirement age

Following HLT, our model period is one year, maximum life length is 80 years, and career length is the number of periods an agent works. This configuration creates lumpiness in an individual's labor supply. Limited heterogeneity among employed workers – four ability groups and two schooling levels – means that lumpiness can also occur in aggregate labor supply outcomes. That potentially complicates computing an equilibrium. Thus, for the

Section 5 tax-and-transfer scheme our algorithm failed to converge to an equilibrium for some tax rates because choices of a significant group of agents between retiring in one year versus a year earlier could not be reconciled with an equilibrium value for the lump-sum transfer. On one hand, they *would* like to retire a year earlier if they could receive the *larger* lump-sum transfer that would be the equilibrium amount if their entire group did *not* retire a year earlier. On the other hand, they *would not* like to retire a year earlier if they were to receive the *smaller* lump-sum transfer that would be the equilibrium amount if the group as a whole had retired a year earlier. While the problem seems to resemble the Section 5.2 computational challenge that we resolved by randomizing, a key difference is that the earlier "tipping points" at which identical agents were indifferent between starkly different lifetime career choices could not have been remedied by simply using a finer model period. However, the present computational challenge is amenable to such an approach.

Interpolation via cubic splines. We use cubic splines to interpolate between the computed points that comprise a curve like those depicted in Figure 9. Our purpose is to approximate an agent's maximal lifetime utility while offering a continuous value of a career length by allowing an agent to work just a fraction of his retirement year. We proceed as follows.

1. Interpolate $V^{\hat{n}}(\cdot)$, the value of retiring at a discrete retirement age $\hat{n}$, using cubic splines.

2. Find the maximum of $V^{\hat{n}}(\cdot)$ in the window $[\hat{n}^* - 1, \hat{n}^* + 1]$, where $\hat{n}^*$ is the optimal discrete retirement age. This local maximum is the optimal continuous retirement age.

3. Given the optimal continuous retirement age, we linearly interpolate the agent's paths of human capital, investment, consumption, and savings.

This is a tractable way to approximate finer sub-periods with respect to an agent's choice of career length while still retaining HLT's annual model period. The strategy enables us to successfully compute equilibria at most tax rates in our experiments except for instances of the Section 5.2 computational challenge, which will be resolved below in Section A.3.2.

Analytical method. For agents who find it optimal not to accumulate human capital on the job, there is an analytical formula for an optimal continuous retirement age that comes directly from the solution to the annual problem. It is exact in the sense that it gives the same retirement age that we would have obtained if we had assumed that career length were a continuous choice variable. Equivalence of a two-stage optimization problem and a single maximization problem sheds light on the workings of our model and its HLT building blocks. In particular, HLT's assumptions of complete markets and CES preferences over consumption imply that the shape of a consumption profile from the first stage will also be optimal in a second stage. The consumption profile just shifts proportionately down or up depending on whether an optimal continuous retirement age is shorter or longer than the annual retirement age from the first stage. The optimal continuous retirement age is then pinned down by the first-order condition with respect to an incremental change in career length around an optimal annual retirement age from the first stage.[35] Hence, we can solve for a continuous retirement age in closed form as follows.

Let the solution to a worker's annual optimization problem be a retirement age $\hat{n}^*$, a final human capital stock H equal to the initial endowment, and a lifetime consumption profile $\{c_n\}_{n=\underline{\eta}}^{\overline{\eta}}$. The worker now gets the opportunity to re-optimize, choosing a continuous retirement age. The re-optimized retirement age should fall in the open interval $(\hat{n}^*-1, \hat{n}^*+1)$ since both end points were found to yield lower lifetime utility than retirement age $\hat{n}^*$ in the annual optimization problem. Let $\hat{n}^* + m$ be the optimal continuous retirement age where $m \in (-1, 1)$.

The re-optimization retains the underlying annual structure of our model. Specifically, for any downward change $m \in (-1, 0]$, the annual discount factor at age $\hat{n}^* - 1$ applies to all choices within this window of change, as well as constant age-dependent efficiency units of human capital. In the case of an upward change $m \in [0, 1)$, we apply the annual discount factor and efficiency units of human capital at age $\hat{n}^*$.

The change m in retirement age alters a worker's after-tax labor income, which we express as a fraction of the present value of the worker's lifetime consumption in terms of goods at

[35] In contrast to the spline interpolation method, the analytical formula applies the proper present-value discount factor associated with the 'annual windows' in which any of the repercussions of the optimal perturbation falls, including changes in consumption over the entire lifespan and, in particular, the marginal change in career length and disutility of working where the proper annual discount factor depends on if the optimal perturbation calls for a shorter or a longer career length.

age $\underline{\eta}$ (i.e., the age at which the worker enters the economy),

$$\Delta^I(m) = \begin{cases} \dfrac{\left[\dfrac{1}{1+(1-\tau_k)r}\right]^{\hat{n}^*-\underline{\eta}-1} (1-\tau_l-\tau_p)mR^S e(\hat{n}^*-1)H}{\sum_{n=\underline{\eta}}^{\overline{\eta}} \left[\dfrac{1}{1+(1-\tau_k)r}\right]^{n-\underline{\eta}} c_n} & m \in (-1,0] \\ \dfrac{\left[\dfrac{1}{1+(1-\tau_k)r}\right]^{\hat{n}^*-\underline{\eta}} (1-\tau_l-\tau_p)mR^S e(\hat{n}^*)H}{\sum_{n=\underline{\eta}}^{\overline{\eta}} \left[\dfrac{1}{1+(1-\tau_k)r}\right]^{n-\underline{\eta}} c_n} & m \in [0,1) \end{cases}$$

Note that this formula, as an expression for the change in disposable income, would have to be augmented under the baseline social security program if any contemplated change in retirement age occurs after the official retirement η_p. Specifically, we would then have to account for an accompanying change in social security benefits collected.

In response to this change in lifetime labor income, the worker wants to change their entire consumption profile proportionally by the same fraction $\Delta^I(m)$. The change in the lifetime utility of consumption becomes:

$$\begin{aligned} \Delta^C(m) &= \sum_{n=\underline{\eta}}^{\overline{\eta}} \delta^{n-\underline{\eta}} \log\big((1+\Delta^I(m))c_n\big) - \sum_{n=\underline{\eta}}^{\overline{\eta}} \delta^{n-\underline{\eta}} \log(c_n) \\ &= \frac{1-\delta^{\overline{\eta}-\underline{\eta}+1}}{1-\delta} \log\big(1+\Delta^I(m)\big) \end{aligned}$$

The change in lifetime disutility of work is:

$$\Delta^B(m) = \begin{cases} \delta^{\hat{n}^*-\underline{\eta}-1} mB & m \in (-1,0] \\ \delta^{\hat{n}^*-\underline{\eta}} mB & m \in [0,1) \end{cases}$$

At the optimal m the marginal disutility of working at a small change in career length should equal the marginal utility of the additional consumption:

$$\frac{d}{dm}\Delta^B(m) = \frac{d}{dm}\Delta^C(m) = \frac{1-\delta^{\overline{\eta}-\underline{\eta}+1}}{1-\delta} \frac{1}{1+\Delta^I(m)} \frac{d}{dm}\Delta^I(m)$$

We can now solve out for the optimal choice of m:

$$m = \begin{cases} \dfrac{1-\delta^{\overline{\eta}-\underline{\eta}+1}}{(1-\delta)\delta^{\hat{n}^*-\underline{\eta}-1}B} - \dfrac{\sum_{n=\underline{\eta}}^{\overline{\eta}} \left[\dfrac{1}{1+(1-\tau_k)r}\right]^{n-\underline{\eta}} c_n}{\left[\dfrac{1}{1+(1-\tau_k)r}\right]^{\hat{n}^*-\underline{\eta}-1} (1-\tau_l-\tau_p)R^S e(\hat{n}^*-1)H} & m \in (-1,0] \\ \dfrac{1-\delta^{\overline{\eta}-\underline{\eta}+1}}{(1-\delta)\delta^{\hat{n}^*-\underline{\eta}}B} - \dfrac{\sum_{n=\underline{\eta}}^{\overline{\eta}} \left[\dfrac{1}{1+(1-\tau_k)r}\right]^{n-\underline{\eta}} c_n}{\left[\dfrac{1}{1+(1-\tau)r}\right]^{\hat{n}^*-\underline{\eta}} (1-\tau_l-\tau_p)R^S e(\hat{n}^*)H} & m \in [0,1) \end{cases}$$

When implemented, after calculating the optimal discrete retirement age, we calculate the two values of n that correspond to downward and upward perturbations of the discrete retirement age. We then check which of the three options yields the highest lifetime utility to arrive at the final continuous retirement age, which corresponds to an interior solution where $m \in (-1, 1)$. That interior solution can sometimes be a corner solution to the two equations above, at $m = 0$.

We can use the analytical formula to check for the accuracy of the spline interpolation method. Under the Section 5 tax-and-transfer scheme, at high enough tax rates every agent eventually chooses not to accumulate human capital on the job. For example, at tax rate $\tau_l = 0.61$, high school workers of all four ability groups are accumulating virtually no human capital on the job in Figure 7. Thus, the analytical formula provides exact optimal continuous retirement ages for these workers that we compare to spline interpolations in Table A.1. Specifically, using equilibrium prices and lump-sum transfers computed from our model with discrete annual retirement ages, we calculate continuous retirement ages with both the spline interpolation method and the analytical formula. The spline interpolations yield approximations very close to the exact continuous retirement ages of the analytical formula, the largest deviation being only half a month. This finding makes us confident to use the spline interpolation method, whereas the analytical formula would not apply for the general case of agents choosing to accumulate additional human capital on the job.

A.3.2 Convexification at career strategy indifferences

Another challenge arises when a given worker type (ability, schooling level) is indifferent between a short career length with low human capital accumulation and a long career length

Table A.1: Continuous retirement ages of different high school worker types using the spline interpolation method and the analytical formula, respectively.

	Discrete	Spline	Analytical
Ability 1	30	29.7507	29.7930
Ability 2	36	35.7252	35.7626
Ability 3	39	38.3935	38.3862
Ability 4	36	36.4830	36.5040

Notes: Evaluated at tax rate 0.61 under the Section 5 tax-and-transfer scheme. We use the equilibrium prices and lump-sum transfer associated with this tax rate in the discrete retirement age solution.

with high human capital accumulation. In this situation, the computational challenge is that retirement ages do not respond smoothly to small changes in prices, and hence it becomes difficult to converge on a set of prices that satisfy the general equilibrium conditions. In these cases, we use a convexification strategy that splits up the worker type who is indifferent between the two career lengths. It works as follows.

1. For a given guess of prices and the lump-sum transfer, we solve the agents' problem as above using the spline approximation method.

2. For each worker type, we check whether the value function over retirement ages has two local maxima

3. We then identify which worker type is "most indifferent" by choosing the type which has the smallest difference between the two local maxima. If this difference is greater than 0.05%, then we do not convexify by splitting up any worker type but proceed to complete the algorithm in the usual way (skip the next step).

4. With the agent who is most indifferent, vary the fraction that retire at each of the two ages until the general equilibrium conditions are satisfied.

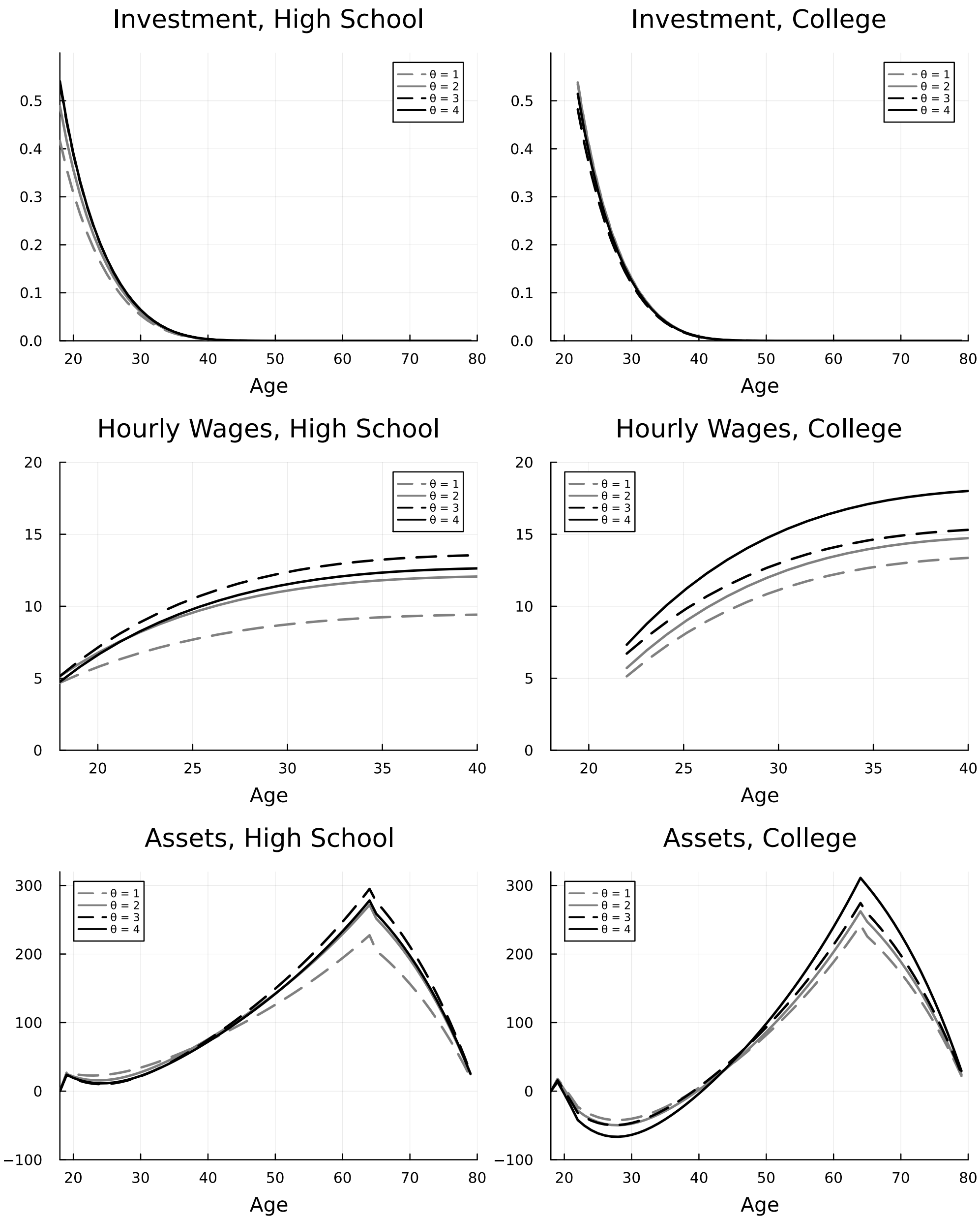
This method introduces smoothness into the general equilibrium problem. This helps us to successfully compute equilibrium prices at tax rates where a certain type of agent is nearly indifferent between two starkly different career lengths. In these situations, as the tax rate rises, the agent type who is indifferent gradually moves from a situation where all of them are choosing the long career length to one where all of them are choosing the short career length.

B Life-cycle profiles in HLT's model and ours

Figure B.1 provides some representative plots from our replication of the HLT model. In the case of human capital investment and wage profiles, these plots line up closely with Figures 1-3 of HLT. Both high school and college-educated workers initially spend around 50% of their time in human capital investment, but this rapidly declines to 0 by around age 45. Given this investment in human capital and the upward sloping profile of wages, all agents save little before they are around 40. Capital holdings then increase rapidly until retirement, after which agents draw down their savings until the end of the life-cycle.

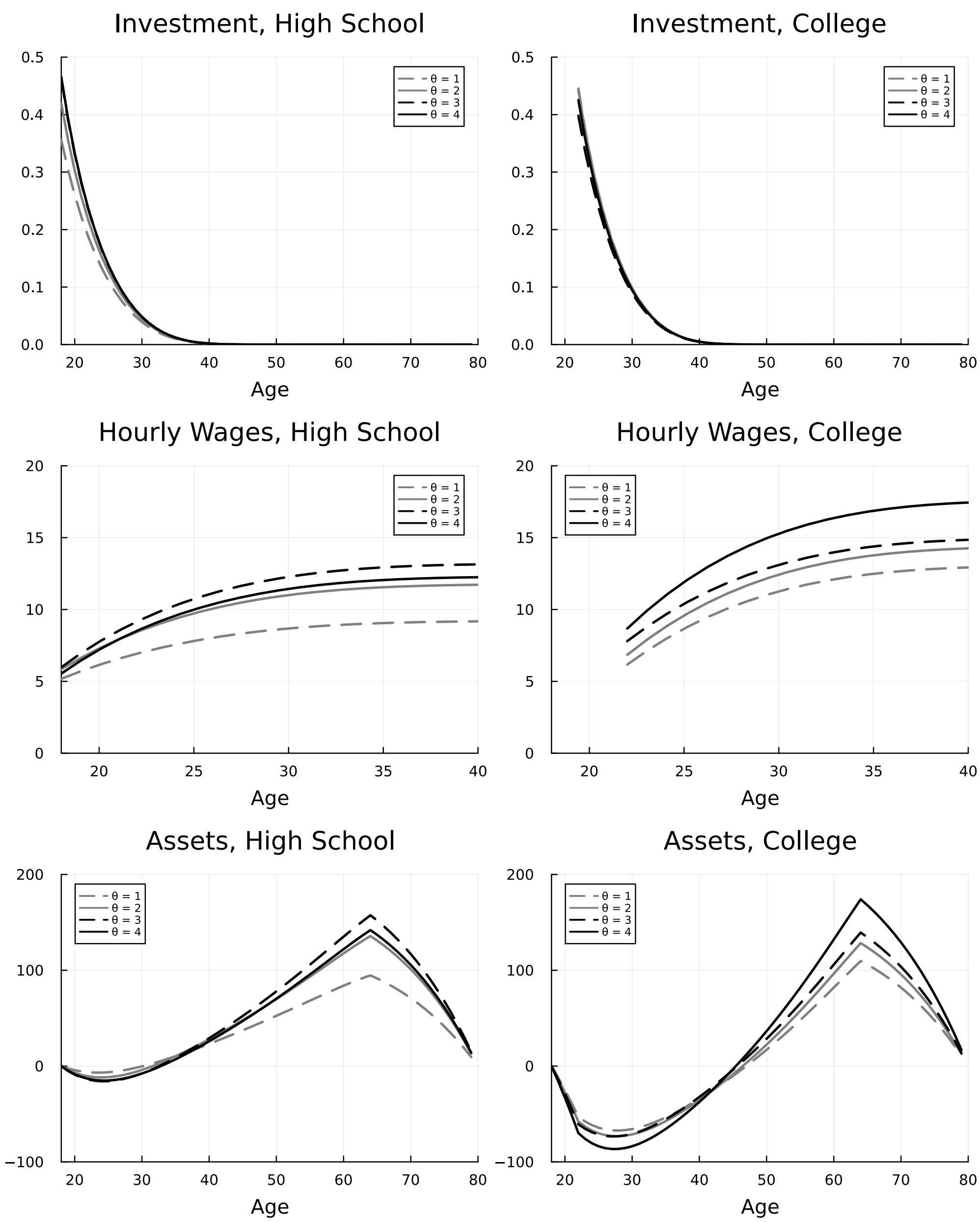
Figure B.2 reports the corresponding life-cycle profiles in our baseline time-averaging version of the HLT model. The profiles in our model are very similar to those in the HLT model in Figure B.1. The only difference is that the levels of savings are significantly lower in our model as compared to the HLT model. This is explained by us having introduced a pay-as-you-go social security system in our model.

Figure B.1: Life-cycle profiles in the HLT model



Notes: The profiles are computed from our replication of the HLT model.

Figure B.2: Life-cycle profiles in our baseline model



C Sensitivity analyses of B and $e(n)$

If workers choose a corner solution of retiring at the official retirement age $\eta_p = 65$, the fixed disutility B of working and the two parameters of the logistic function for efficiency units of human capital, ϕ_1 and ϕ_2, are not well pinned down under our baseline social security system. Furthermore, in countries like the U.S. that have moved from earlier social security arrangements that resembled our baseline social security system to reformed arrangements that more nearly resemble our stylized social security reform, it is possible that transitions are incomplete. For these and related reasons, rather than seeking to infer these parameters from U.S. data, we confine our inquiry to a sensitivity analysis of the fixed disutility of working and the decline of efficiency units of human capital in old age.

Based on HLT's and our assumption of an 80-year lifespan, we set an inflection point $\phi_2 = 75$ for the Section 3.2 logistic function that describes efficiency units of human capital, so that efficiency units of human capital will have declined by 50 percent at the age of 75. That makes all agents want to retire during their lifetimes under plausible parameterizations of the fixed disutility of working. While holding $\phi_2 = 75$ constant, we focus here on alternative configurations of the fixed disutility B of working and the "slope" coefficient ϕ_1 of the logistic function, keeping in mind that our calibration target is that all agents choose to retire at the official retirement age 65 in the baseline steady state.

Given our baseline parameterization of the social security system and $\phi_2 = 75$, we investigate the first Section 3.2 calibration step in the following way. For each value of the fixed disutility B of working, we compute a range of slope coefficients ϕ_1 of the logistic function that attain our calibration target that all agents choose to retire at the official retirement age 65. As we perturb toward either a lower or a higher value of B, the width of the range of permissible values of ϕ_1 eventually starts to shrink, and at some point becomes zero because the value of the fixed disutility B of working has become so low or so high that no value of ϕ_1 lets us attain the calibration target. Instead, for any value of ϕ_1, there will be at least one category of workers, defined by their ability group and schooling choice, who want to retire either later or earlier than the official retirement age 65.

Searching over (B, ϕ_1) in this way yields approximate end-point coordinates of $(0.59, 0.09)$ and $(0.9, 0.31)$, respectively. Our baseline parameterization was actually guided by outcomes of this search process when we selected an intermediate pair (B, ϕ_1) subject to the additional constraint that a noticeable decline in efficiency units of human capital should not occur until workers are in their 60's. Accordingly, in Section 3.2 we picked $\phi_1 = 0.2$ with the corresponding age profile of the conversion of one unit of human capital into efficiency units depicted in Figure 1, along with profiles for the two 'end-point' values of $\phi_1 = 0.09$ and $\phi_1 = 0.31$, respectively. Associated with parameter value $\phi_1 = 0.2$ is a range of the fixed disutility of working, $B \in (0.77, 0.84)$, that makes all agents choose to retire at age 65; we selected the midpoint $B = 0.8$ for our baseline parameterization. We now examine how our analyses would have changed if we had instead adopted one of the two pairs of end-point coordinates for the parameter pair (B, ϕ_1).

Tables C.1 and C.2 report steady states that we would have attained if we had let parameters (B, ϕ_1) be 'low' at $(0.59, 0.09)$ or 'high' at $(0.9, 0.31)$, while retaining the other calibration steps in Section 3.2. These alternative calibrations change little overall.[36] For alternative initial steady states and future steady states after skill-biased technological change, outcomes in Table C.1 are very similar to those of our parameterization in the last two columns of Table 3 because agents continue to retire at the official retirement age 65 with one exception: with 'high' (B, ϕ_1), high school workers of the lowest ability group 1 retire a couple of years earlier in the steady state after skill-biased technological change. This equilibrium change foreshadows outcomes under social security reform. Thus, as compared to our steady state after social security reform in the second column of Table 5, all high school workers retire 1-3 years *earlier* with 'high' (B, ϕ_1), while they retire 1.5-2 years *later* with 'low' (B, ϕ_1) in Table C.2. In both cases, college workers retire about one year later than in our steady state after social security reform.

To offer perspective on effects of different values of the fixed disutility B of working, consider a single worker who enters our baseline economy with his own value of $\hat{B} \in [0.3, 1.3]$. Figure C.1(a) shows the optimal retirement age as a function of $\hat{B}$ for a single high school worker of the lowest ability group 1 that contributes most to the pool of high school workers. The solid and dashed lines depict retirement ages conditional on that agent having the baseline logistic function for the efficiency units of human capital and instead of experiencing no depreciation, respectively. Figure C.1(b) is the corresponding graph for a single college worker of the highest ability group 4 that contributes the most to the pool of college workers. Since we consider a single individual who enters the economy with these different characteristics, that worker's arrival bring no effects on aggregate outcomes, so that the interest rate

[36]The only substantive difference is that the aggregate stock of physical capital for 'low' (B, ϕ_1) increases much more in response to skill-biased technological change as compared to the other two economies. The explanation has to do with the coefficient $\phi_1 = 0.09$ that causes efficiency units of human capital to decline significantly earlier in life as shown in Figure 1, which creates a larger demand for lifecycle savings among workers whose labor earnings taper off much earlier in life.

Table C.1: Robustness: HLT Experiment: Skill-Biased Technical Change

	Original Calibration		Low ϕ_1/Low B		High ϕ_1/High B	
	Baseline	SBTC	Baseline	SBTC	Baseline	SBTC
r	0.0588	0.0599	0.0588	0.0602	0.0588	0.0597
R^1	2	2.27	2	2.26	2	2.27
R^2	2	2.45	2	2.45	2	2.45
Utilized Human Capital $\bar{H}_1$	249	94	225	85	253	94
Utilized Human Capital $\bar{H}_2$	287	459	258	413	292	464
Physical Capital $\bar{K}$	5605	6849	5047	9203	5703	6263
College Attendance ($\theta = 1$)	0.11	0.47	0.11	0.47	0.11	0.46
College Attendance ($\theta = 2$)	0.34	0.77	0.34	0.77	0.34	0.76
College Attendance ($\theta = 3$)	0.56	0.90	0.56	0.90	0.56	0.90
College Attendance ($\theta = 4$)	0.86	0.99	0.86	0.99	0.86	0.99
	HS/College	HS/College	HS/College	HS/College	HS/College	HS/College
Retirement Age ($\theta = 1$)	65,65	65,65	65,65	65,65	65,65	62.7,65
Retirement Age ($\theta = 2$)	65,65	65,65	65,65	65,65	65,65	65,65
Retirement Age ($\theta = 3$)	65,65	65,65	65,65	65,65	65,65	65,65
Retirement Age ($\theta = 4$)	65,65	65,65	65,65	65,65	65,65	65,65

Notes: θ denotes the four ability types. $\bar{K}$ measures total capital (held by agents and "investors").

Table C.2: Robustness: Social Security Reform

	Original Calibration		Low ϕ_1/Low B		High ϕ_1/High B	
	Baseline	Reform	Baseline	Reform	Baseline	Reform
r	0.0588	0.0617	0.0588	0.0617	0.0588	0.0604
R^1	2	1.97	2	1.97	2	2
R^2	2	1.97	2	1.97	2	1.97
Utilized Human Capital $\bar{H}_1$	249	258	225	237	253	257
Utilized Human Capital $\bar{H}_2$	287	301	258	274	292	305
Physical Capital $\bar{K}$	5605	5583	5047	7514	5703	5072
College Attendance ($\theta = 1$)	0.11	0.08	0.11	0.09	0.11	0.07
College Attendance ($\theta = 2$)	0.34	0.29	0.34	0.30	0.34	0.26
College Attendance ($\theta = 3$)	0.56	0.50	0.56	0.52	0.56	0.47
College Attendance ($\theta = 4$)	0.86	0.83	0.86	0.84	0.86	0.80
	HS/College	HS/College	HS/College	HS/College	HS/College	HS/College
Retirement Age ($\theta = 1$)	65,65	63.3,70.9	65,65	65.3,72.2	65,65	59.9,71.8
Retirement Age ($\theta = 2$)	65,65	64.1,70.9	65,65	65.9,72.1	65,65	61.9,71.8
Retirement Age ($\theta = 3$)	65,65	64.6,70.6	65,65	66.1,72.0	65,65	63.2,71.5
Retirement Age ($\theta = 4$)	65,65	64.5,70.8	65,65	66.1,72.1	65,65	63.5,71.7

Notes: θ denotes the four ability types. $\bar{K}$ measures total capital (held by agents and "investors").

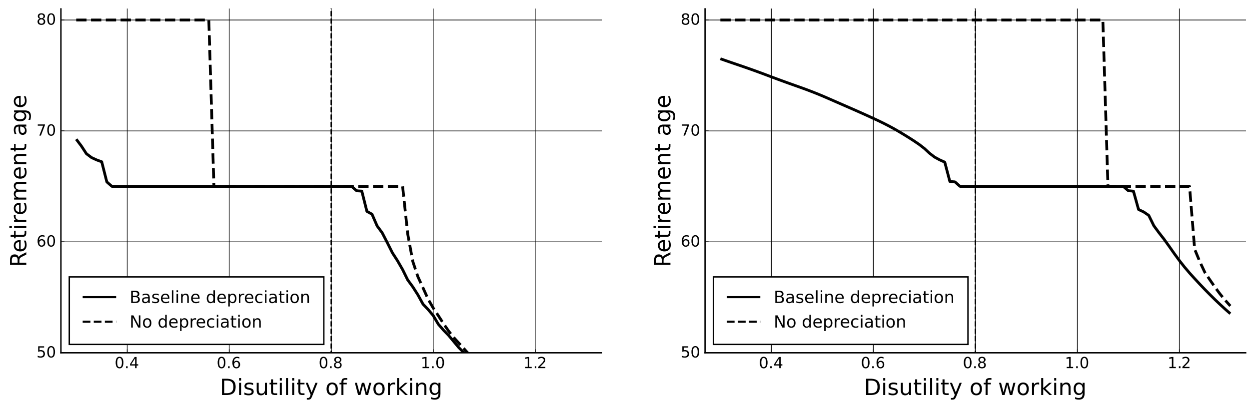


Figure C.1: Retirement age of a single agent in the baseline economy as a function of his "perturbed" disutility of working, $\hat{B}$. The individual is either a high school worker of ability group 1 (Panel A) or a college worker of ability group 4 (Panel B), who is subject to either baseline depreciation of human capital (solid line) or no depreciation (dashed line).

and prices of human capital stay at their baseline steady state values.

At sufficiently low values of $\hat{B}$, all of the individuals in Figure C.1 work beyond the official retirement age 65 and, if there is no depreciation of human capital, both high school and college graduates work their entire lifespans until age $\overline{\eta} = 80$. As we gradually increase the fixed disutility of work, over some interval for $\hat{B}$ agents eventually get stuck at the corner and retire at age 65. At the baseline parameter value $\hat{B} = B = 0.8$, points on a solid line depict a representative high school worker of ability group 1 and a representative college worker of ability group 4, respectively, in our baseline economy. For reasons explained in Sections 4 and 5, a high school worker is more likely than a college worker to shorten his career length. This is reflected here in a representative high school worker of ability group 1 being closer to the right end of the interval of $\hat{B}$ over which it is optimal to retire at age 65 than is a representative college worker of ability group 4, i.e., it takes a smaller increase in $\hat{B}$ to induce the high school worker in Figure C.1(a) to retire earlier than age 65 as compared to the college worker in Figure C.1(b). Turning to corresponding flat segments of the dashed lines for the two individuals whose human capitals don't depreciate, there is no overlap of the two segments, so there is no common value of $\hat{B}$ that would make both the high school and the college worker choose to retire at the official retirement age 65.

Sharp drops in retirement ages in Figure C.1 and accompanying sharp drops in the end-of-life human capitals stock (not shown) are manifestations of Section 5.2 nonconvexities in the space of career strategies, but now they show up in responses to perturbations of an agent's disutility $\hat{B}$ of work instead of to our earlier perturbations of a tax rate. Effects are especially striking when human capital does not depreciate (the dashed lines) and an agent switches from working his entire life and instead retires at the official retirement age 65. Besides intrinsic features of the Ben-Porath human capital technology that can put nonconvexities into the problem of choosing a career strategy, an additional factor affects transitions from working an entire life-time to retiring at age 65. Thus, under the baseline social security system, someone who works beyond age 65 sacrifices the present value of his foregone social security benefits, so that an implicit extra tax wedge that induces all workers in the baseline economy to retire at age 65 now also awards a large capital gain to anyone who retires at age 65 instead of working until 80: by making that downward jump in retirement age, the worker in one swoop can collect the present value of social security benefits between ages 65 and 80. In contrast, for agents who confront the baseline depreciation of human capital in Figure C.1 (solid lines), the lure of that capital gain is diminished by an improved conversion of human capital into efficiency units as an agent "walks up" along the schedule in Figure 1 (solid line) so that the decreases in the retirement age toward the official retirement age becomes more gradual. Eventually, as optimal retirement ages continue to decline at the far right ends of Figure C.1, the solid and dashed lines for a worker converge so that it no longer matters whether human capital depreciates since the rate of conversion of human capital into efficiency units approaches being one-to-one at the chosen retirement age.

D Varying the generosity of social security benefits

The social security reform of Section 4 considers a reform which allows workers to receive social security benefits from age 65 regardless of whether or not they have retired. In this section we provide further details on the alternative social security reform detailed in footnote 23. This is a reform where we retain the assumption that workers are still only able to receive a pension from 65 if they have stopped working, and instead vary the generosity of the pension, P.

For each pension level considered, we solve for the equilibrium levels of R^1, R^2, and r under the baseline parameterization including baseline tax rates. As in our other policy experiments in Sections 4–6, the government budget constraint is satisfied by adjusting government consumption G. We consider a range of P from 0 to 20, where the highest level is approximately equal to average labor earnings.

Figure D.1 plots the equilibrium retirement ages for each schooling level and ability type. As benefits are raised above the baseline level, college educated workers continue to retire

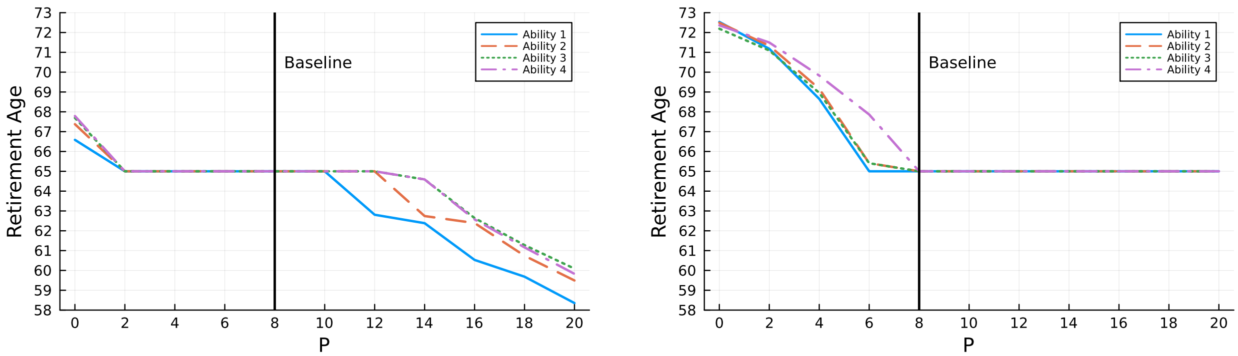


Figure D.1: Retirement ages of high school-educated workers (Panel A) and college-educated workers (Panel B) by ability groups as functions of the social security benefit P. For each value of P, equilibrium prices $\{R^1, R^2, r\}$ are computed under the baseline parameterization including baseline tax rates. The government budget constraint is satisfied by adjusting government consumption G.

at 65, while high-school educated workers begin to retire earlier and earlier. The fact that college educated workers continue to retire at 65, despite significant increases in pension generosity, is evidence that these are the workers most constrained by the kink in the budget constraint implied by the implicit tax contained in the social security system.

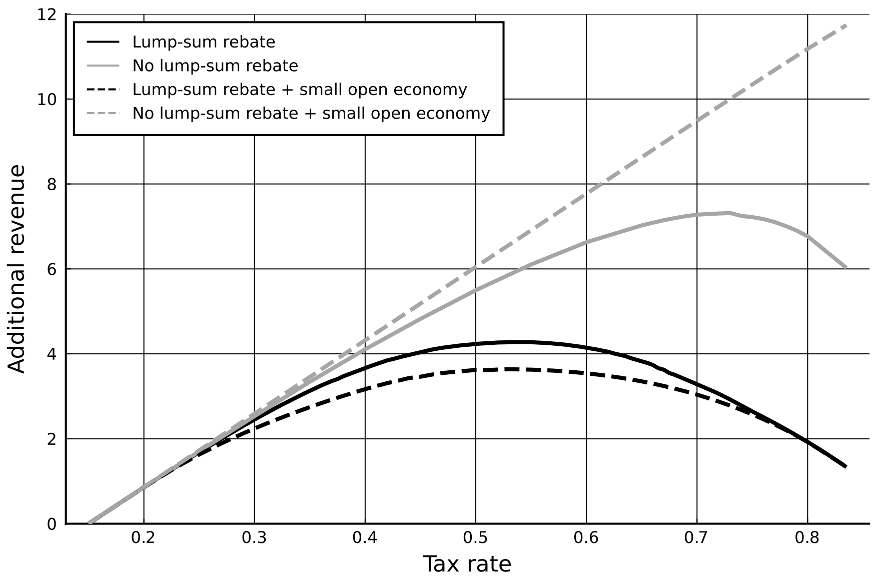
This kink in budget constraints is weakened as P is lowered. For low enough levels of P, college educated workers become willing to continue working past 65, giving up the social security benefits from 65 until their retirement age. When P reaches zero, the kink is completely removed, and high school educated workers also retire after 65. Note, this particular experiment does not exactly correspond to the "no social security" baseline in Table 5 of Section 4, as here we are not recalibrating the fraction of capital held by agents in our model in order to maintain a capital-output ratio of 4.

E Tax Experiments in the Small-Open Economy

In this section, we provide a more comprehensive account of how some of the aggregate outcomes we study in Section 5.1 change in the small-open version of our economy with lump-sum rebates handed back to workers.

Figure E.1 shows the same Laffer curves as in Figure 2(a) from Section 5.1, with the addition of the small-open version of our economy where the tax revenues are given back to households (black dashed line). In the small-open version, where the interest rate is held

Figure E.1: Laffer curves



fixed at its baseline rate, the tax revenues are also hump-shaped but lower, particularly in the middle-range of tax rates. Overall, we find that many of the patterns from the general equilibrium with lump-sum rebates carry over to the small-open version of the economy, but occur at earlier tax rates. In the small-open economy, the suppression of labor supply occurs earlier, which is why tax revenues peak at a lower level. The reasons for this will be come clearer from the next set of figures.

Figure E.2 shows the average retirement ages for both high school and college workers. When tax revenues are handed back to households, the small-open economy yields changes that go in the opposite direction to the scenario without lump-sum rebates. High-school and college workers both begin retiring earlier at lower tax rates. This is a cause of the fall in labor supply which leads to the fall in revenue in Figure E.1. From Figure E.2, we can see that below tax rates of 0.5, this is driven primarily by the shortening of career lengths of high school workers, whereas at higher tax rates, it is driven by the same effect among college workers.

Figure E.2: Retirement ages of high school workers (Panel A) and college workers (Panel B)

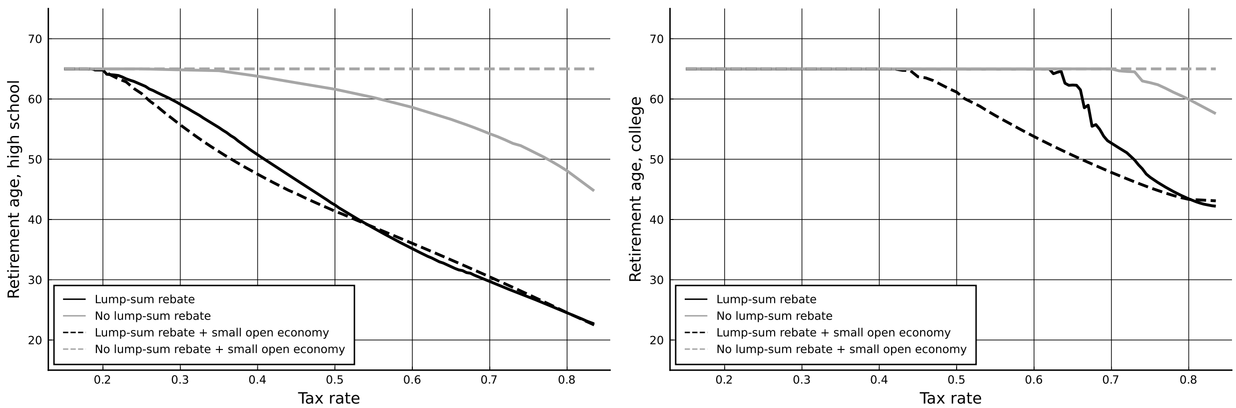
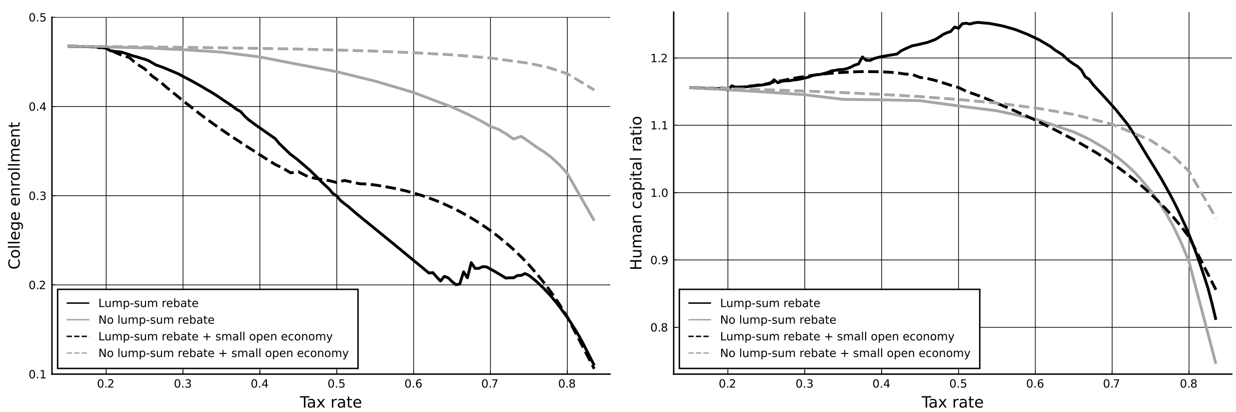


Figure E.3: College enrollment (Panel A) and ratios of college to high school human capital in the production of goods (Panel B)



These shortened career lengths in this economy are also accompanied by an earlier fall in the college enrollment rates, shown as the black dashed line in Figure E.3(a). In fact, the entire path is similar to the economy where the interest rate is allowed to adjust – with an initial decline, a flattening, and then another decline – but shifted to the left. The dashed line in Figure E.3(b) shows the corresponding pattern in the ratio of college to high school human capital. Like in the closed economy, it rises and then falls, except in the open economy the rise is much more muted and the fall occurs earlier. The same is true for the skill premium in Figure E.4: the changes go in the same direction as the closed economy but are muted and start at lower tax rates.

Figure E.4: Relative price of college to high school human capital